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Introduction

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1.1. ENERGY AND POWER

Energy. The historical as well as present-day civilization of mankind are closely interwoven with energy, and there is little reason to doubt but that in the future our existence will be ever more dependent upon this thing called **energy.** *Energy* probably was the original stuff or creation. Energy appears in many forms, but has one thing in common—*energy is possessed of the ability to produce a dynamic, vital effect.* Energy is associated with physical substance, but is not a substance itself. It shows itself by excited, animated state assumed by material which receives energy. The examples which can be quoted in respect of energy are almost endless in number. In this era energy is being produced in enormous quantities. As more and more of it is produced per person, the comforts, conveniences, and pleasures of life are even enhanced.

Energy exists in various forms *e.g. mechanical, thermal, electrical* etc. One form of energy can be converted into other by the use of suitable arrangements. Out of all these forms of energy, *electrical energy* is *preferred* due to the following *advantages* : 1. *Can be easily transported from one place to another.* 2. *Losses in transport are minimum.* 3. *Can be easily subdivided.* 4. *Economical in use.* 5. *Easily converted into other forms of energy.* 6. *Easily controlled and regulated to suit requirements.*

Power. Any physical unit of energy when divided by a unit of time automatically becomes a *unit of power.* However, it is in connection with the mechanical and electrical forms of energy that the term "*power*" is generally used. The rate of production or consumption of heat energy and, to a certain extent, of radiation energy is not ordinarily thought of as power. *Power is primarily associated with mechanical work and electrical energy*. Therefore, *power* can be defined as the *rate of flow of energy* and can state that a *power plant* is a *unit built for production and delivery of a flow of mechanical and electrical energy.* In common usage, a machine or assemblage of equipment that produces and delivers a flow of mechanical or electrical energy is a *power plant.* Hence, an internal combustion engine is a power plant, a water wheel is a power plant, etc. However, what we generally mean by the term is that assemblage of equipment, permanently located on some chosen site which receives raw energy in the form of a substance capable of being operated on in such a way as to produce electrical energy for delivery from the power plant.

With the advancement in technology the power consumption is rising steadily. This necessitates that in addition to the existing sources of power such as coal, water, petroleum etc. other sources of energy should be searched out and new and more efficient ways of producing energy should be devised. Nuclear energy has enlarged the world's power resources. The energy released by *1 kg of uranium is equivalent to energy obtained by 4500 tonnes of high grade coal.*

1.2. SOURCES OF ENERGY

The various sources of energy are :

1. Fuels
    - Solids—Coal, coke anthracite etc.
    - Liquids—Petroleum and its derivates
    - Gases—Natural gas blast furnace gas etc.
2. Energy stored in water
3. Nuclear energy
4. Wind power
5. Solar energy
6. Tidal power
7. Geothermal energy
8. Thermoelectric power.

1.2.1. Fuels

Fuels may be chemical or nuclear. Here we shall consider chemical fuels only.

A chemical fuel is a substance which releases heat energy on combustion. The principal combustible elements of each fuel are *carbon* and *hydrogen*. Though *sulphur* is a combustible element too but its presence in the fuel is considered to be *undesirable.*

Classification of fuels :

Fuels can be *classified* according to whether

1. they *occur in nature* called *primary fuels* or are *prepared* called *secondary fuels.*

2. they are *in solid, liquid* or *gaseous state.* The detailed classification of fuels can be given in a summary form as below :

Type of fuel	*Natural (Primary)*	*Prepared (Secondary)*
Solid	Wood	Coke
	Peat	Charcoal
	Lignite coal	Briquettes
Liquid	Petroleum	Gasoline
		Kerosene
		Fuel oil
		Alcohol
		Benzol
		Shale oil
Gaseous	Natural gas	Petroleum gas
		Producer gas
		Coal gas
		Coke-oven gas
		Blast furnace gas
		Carburetted gas
		Sewer gas

1.2.1.1. Solid Fuels

Coal. Its main constituents are carbon, hydrogen, oxygen, nitrogen, sulphur, moisture and ash. Coal passes through different stages during its formation from vegetation. These stages are enumerated and discussed below :

Plant debris-Peat-Lignite-Brown coal-Sub-bituminous coal-Bituminous coal-Semi-bituminous coal-Semi anthracite coal-Anthracite coal-graphite.

Peat. It is the first stage in the formation of coal from wood. It contains huge amount of moisture and therefore it is dried for about 1 to 2 months before it is put to use. It is used as a domestic fuel in Europe and for power generation in Russia. In India it does not come in the categories of good fuels.

Lignites and brown coals. These are intermediate stages between peat and coal. They have a woody or often a clay like appearance associated with high moisture, high ash and low heat contents. Lignites are usually amorphous in character and impose transport difficulties as they break easily. They burn with a smoky flame. Some of this type are suitable for local use only.

Bituminous coal. It burns with long yellow and smoky flames and has high percentages of volatile matter. The average calorific value of bituminous coal is about 31350 kJ/kg. It may be of two types, namely *caking* or *non-caking*.

Semi-bituminous coal. It is softer than the anthracite. It burns with a very small amount of smoke. It contains 15 to 20 per cent volatile matter and has a tendency to break into small sizes during storage or transportation.

Semi-anthracite. It has less fixed carbon and less lustre as compared to true anthracite and gives out longer and more luminous flames when burnt.

Anthracite. It is very hard coal and has a shining black lustre. It ignites slowly unless the furnace temperature is high. It is noncaking and has high percentage of fixed carbon. It burns either with very short blue flames or without flames. The calorific value of this fuel is high to the tune of 35500 kJ/kg and as such is very suitable for steam generation.

Wood charcoal. It is obtained by destructive distillation of wood. During the process the volatile matter and water are expelled. The physical properties of the residue (charcoal) however depends upon the rate of heating and temperature.

Coke. It consists of carbon, mineral matter with about 2% sulphur and small quantities of hydrogen, nitrogen and phosphorus. It is solid residue left after the destructive distillation of certain kinds of coals. It is smokeless and clear fuel and can be produced by several processes. It is mainly used in blast furnace to produce heat and at the same time to reduce the iron ore.

Briquettes. These are prepared from fine coal or coke by compressing the material under high pressure.

Analysis of coal. The following two types of analysis is done on the coal :

1. Proximate analysis
2. Ultimate analysis.

1. **Proximate analysis.** In this analysis, *individual elements are not determined ; only the percentage of moisture, volatile matters, fixed carbon and ash are determined.*

Example. Moisture = 4.5%, volatile matter = 5.5%, fixed carbon = 20.5%.

This type of analysis is easily done and is for *commercial purposes* only.

2. **Ultimate analysis.** In the ultimate analysis, the percentage of various elements are determined.

Example. Carbon = 90%, hydrogen = 2%, oxygen = 4%, nitrogen = 1%, sulphur = 15% and ash = 1.5%.

This type of analysis is useful for *combustion calculations.*

Properties of Coal. *Important properties of coal* are given below :

1. Energy content or heating value
2. Sulphur content
3. Burning characteristics
4. Grindability
5. Weatherability
6. Ash softening temperature.

A good coal should have :

(*i*) low ash content and high calorific value

(*ii*) small percentage of sulphur (less than 1%)

(*iii*) good burning characteristics (*i.e.* should burn freely without agitation) so that combustion will be complete

(*iv*) high grindability index (in case of ball mill grinding)

(*v*) high weatherability.

Ranking of Coal. ASME and ASTM have accepted a specification based on the fixed carbon and heating value of the mineral matter free analysis.

—**Higher ranking** is done on the basis of fixed carbon percentage (*dry basis*).

—**Lower ranking** is done on the heating value on the *moist basis*.

Example. 62% C and a calorific value of 5000 kcal/kg is ranked as (62—500) rank.

Rank is an inherent property of the fuel depending upon its relative progression in the classification process.

Grading of Coal. Grading is done on the following basis :

(*i*) Size (*ii*) Heating value

(*iii*) Ash content (*iv*) Ash softening temperature

(*v*) Sulphur content.

Example. A grade written as 5—10 cm, 500-A8-F24-S1.6 indicate the coal as having :

—a size of 5–10 cm,

—heating value of 5000 kcal/kg,

—8 to 10% ash,

—ash softening temperature of 2400—2590¢ F, and

—a sulphur content of 1.4 to 1.6%.

A rank and grade of a coal gives a complete report of the material. Thus the following rank and grade of the coal described above :

(62—500), 5—10 cm, 500-A8-F24-S1.6.

1.2.1.2. Liquid Fuels

The chief source of liquid fuels is petroleum which is obtained from wells under the earth's crust. These fuels have proved *more advantageous in comparison to sold fuels* in the following respects.

Advantages :

1. Require less space for storage.
2. Higher calorific value.
3. Easy control of consumption.
4. Staff economy.

5. Absence of danger from spontaneous combustion.
6. Easy handling and transportation.
7. Cleanliness.
8. No ash problem.
9. Non-deterioration of the oil in storage.

Petroleum. There are different opinions regarding the origin of petroleum. However, now it is accepted that petroleum has originated probably from organic matter like fish and plant life etc., by bacterial action or by their distillation under pressure and heat. It consists of a mixture of gases, liquids and solid hydrocarbons with small amounts of nitrogen and sulphur compounds. In India the main sources of petroleum are Assam and Gujarat.

Heavy fuel oil or crude oil is imported and then refined at different refineries. The refining of crude oil supplies the most important product called *petrol*. Petrol can also be made by polymerization of refinery gases.

Other liquid fuels are kerosene, fuels oils, colloidal fuels and alcohol.

The following table gives *composition* of some common liquid fuels used in terms of the elements in *weight percentage.*

Fuel	*Carbon*	*Hydrogen*	*Sulphur*	*Ash*
Petrol	85.5	14.4	0.1	—
Benzene	91.7	8.0	0.3	—
Kerosene	86.3	13.6	0.1	—
Diesel oil	86.3	12.8	0.9	—
Light fuel oil	86.2	12.4	1.4	—
Heavy fuel oil	88.3	9.5	1.2	1.0

Important Properties of Liquid Fuels

(1) Specific gravity
(2) Flash point
(3) Fire point
(4) Volatility
(5) Pour point
(6) Viscosity
(7) Carbon residue
(8) Octane number
(9) Cetane number
(10) Corrosive property
(11) Ash content
(12) Gum content
(13) Heating value
(14) Sulphur content

The requisite properties vary from device to device which uses the fuel to generate power. For example, *higher the octane number, higher can be the compression ratio and the thermal efficiency will be higher.* Similarly, *the cetane number of a diesel oil should be as high as possible.*

In general the liquid fuels should have :

(*i*) low ash content
(*ii*) high heating value
(*iii*) low gum content
(*iv*) less corrosive tendency
(*v*) low sulphur content
(*vi*) low pour point.

Viscosity and other properties vary from purpose to purpose to which the fuel is employed.

1.2.1.3. Gaseous Fuels

Natural gas. The main constituents of natural gas are methane (CH_4) and ethane (C_2H_6). It has calorific value nearly 21000 kJ/m^3. Natural gas is used alternately or simultaneously with oil for internal combustion engines.

Coal gas. Mainly consists of hydrogen, carbon monoxide and hydro-carbons. It is prepared by carbonisation of coal. It finds its use in boilers and sometimes used for commercial purposes.

Coke-oven gas. It is obtained during the production of coke by heating the bituminous coal. The volatile content of coal is driven off by heating and major portion of this gas is utilised in heating the ovens. This gas must be thoroughly filtered before using in gas engines.

Blast furnace gas. It is obtained from smelting operation in which air is forced through layers of coke and iron ore, the example being that of pig iron manufacture where this gas is produced as by product and contains about 20% carbon monoxide (CO). After filtering it may be blended with richer gas or used in gas engines directly. The heating value of this gas is very low.

Producer gas. It results from the partial oxidation of coal, coke or peat when they are burnt with an insufficient quantity of air. It is produced in specially designed retorts. It has low heating value and in general is suitable for large installations. It is also used in steel industry for firing open hearth furnaces.

Water or Illuminating gas. It is produced by blowing steam into white hot coke or coal. The decomposition of steam takes place liberating free hydrogen and oxygen in the steam combines with carbon to form carbon monoxide according to the reaction :

$$C + H_2O \longrightarrow CO + H_2$$

The gas composition varies as the hydrogen content if the coal is used.

Sewer gas. It is obtained from sewage disposal vats in which fermentation and decay occur. It consists of mainly marsh gas (CH_4) and is collected at large disposal plants. It works as a fuel for gas engines which is turn drive the plant pumps and agitators.

Gageous fuels are becoming popular because of following *advantages* they possess :

Advantages :

1. Better control of combustion.
2. Much less excess air is needed for complete combustion.
3. Economy in fuel and more efficiency of furnace operation.
4. Easy maintenance of oxidizing or reducing atmosphere.
5. Cleanliness.
6. No problem of storage if the supply is available from public supply line.
7. The distribution of gaseous fuels even over a wide area is easy through the instrumentality of pipe lines and as such handling of the fuel is altogether eliminated. Gaseous fuels give economy of heat and produce higher temperatures as they can be preheated in regenerative furnaces and thus heat from hot flue gases can be recovered.

Important Properties of Gaseous Fuels

1. Heating value or calorific value
2. Viscosity
3. Specific gravity
4. Density
5. Diffusibility.

Typical composition of some gaseous fuels is given below :

Fuel	H_2	CO	CH_4	C_2H_4	C_2H_6	C_4H_8	O_2	CO_2	N_2
Natural gas	—	1	93	—	3	—	—	—	3
Coal gas	53.6	9.0	25	—	—	3	0.4	3	6
Blast furnace gas	2	27	—	—	—	—	—	11	—

1.2.1.4. Calorific or Heating Values of Fuels

The calorific value of the fuel is defined as the energy liberated by the complete oxidation of a unit mass or volume of a fuel. It is expressed in kJ/kg for solid and liquid fuels and kJ/m^3 for gases.

Fuels which contain hydrogen have two calorific values, the *higher* and the *lower*. The *'lower calorific value'* is the heat liberated per kg of fuel after deducting the heat necessary to vaporise the steam, formed from hydrogen. The *'higher or gross calorific value' of the fuel is one indicated by a constant-volume calorimeter in which the steam is condensed and the heat of vapour is recovered.*

The lower or net calorific value is obtained by subtracting latent heat of water vapour from gross calorific value. In other words, the relation between lower calorific value (L.C.V.) and higher calorific value (H.C.V.) can be expressed in the following way :

$$\text{L.C.V.} = (\text{H.C.V.} — 2465\, m_w) \qquad ...(1.1)$$

where m_w is the mass of water vapour produced by combustion of 1 kg of fuel and 2465 kJ/kg is the latent heat corresponding to standard temperature (saturation) of 15°C.

[**In MKS units:**

$$\text{L. C. V.} = (\text{H. C. V.} - 588.76\, m_w)$$

where m_w is the mass of water vapour produced by combustion of 1 kg of fuel and 588.76 is the latent heat value in kcal as read from steam tables for 1 kg of water vapour]

Dulong's Formula (Solid/liquid fules). Dulong suggested a formula for the calculation of the calorific value of the solid or liquid fuels from their chemical composition which is as given below :

Gross calorific value

or
$$\text{H.C.V.} = \frac{1}{100}\left[33800 + 144000\left(\text{H} - \frac{\text{O}}{8}\right) + 9270\,\text{S}\right] \text{ kJ/kg} \qquad ...(1.2)$$

[**In MKS units:**

$$\text{H.C.V.} = \frac{1}{100}\left[8080\,\text{C} + 34500\left(\text{H} - \frac{\text{O}}{8}\right) + 2240\,\text{S kcal/kg}\right]$$
]

where C, H, O and S are carbon, hydrogen, oxygen and sulphur in percentages respectivily in 100 kg of fuel. In the above formula, the oxygen is assumed to be in combination with hydrogen and only extra surplus hydrogen supplies the necessary heat.

1.2.2. Energy Stored in Water

The energy contained in flowing streams of water is a form of mechanical energy. It may exist as the kinetic energy of a moving stream or as potential energy of water at some elevation with respect to a lower datum level, an example of which would be the water held behind a dam. *Hydraulic plants* are slowly increasing in number, although the number of new plants of this type built is quite small compared with those which exploit heat energy. As a usual thing, the most desirable hydroelectric sites are the first to be utilized, consequently, as more hydroelectric plants are built, the owners must pay increasingly higher development costs.

From the stand point of capitalistic economics, it is often hard to justify the development of hydroelectric power in comparison with steam power, but from the stand point of the conservation of a fixed natural resource, namely, its mineral fuels, it is obvious that every effort should be made to harness the water power of the country, since if unharnessed it goes to waste, whereas fuel, if unmined, remains intact and undiminished in value in the ground.

Water power is quite cheap where water is available in abundance. Although *capital cost of hydroelectric power plants is higher as compared to other types of power plants yet their operating costs are quite low.*

1.2.3. Nuclear Energy

One of the outstanding facts about nuclear power is the *large amount of energy that can be released from a small mass of active material.* Complete fission of one kg of uranium contains the energy equivalent of 4500 tonnes of coal or 2000 tonnes of oil. The *nuclear power is not only available in abundance but it is cheaper than the power generated by conventional sources.*

The following factors go in favour of nuclear energy :

(*i*) Practically independent of geographical factors.

(*ii*) No combustion products.

(*iii*) Clean source of power which does not contribute to air pollution.

(*iv*) Fuel transportation networks and large storage facilities not required.

The economic advantage of nuclear power can be realised only if one can ensure a guaranteed base load of about 75%. The number of electro-chemical processes (fertiliser plants), desalination of water and use of electricity for pumping water from tube wells assure a constant base load. Therefore, such type of power requirements must be developed before the adoption of nuclear power in the country.

1.2.4. Wind Power

The man has been served by the power from winds for many centuries but the total amount of energy generated in this manner is small. The expense of installation and variability of operation have tended to limit the use of the windmill to intermittent services where *its variable output has no serious disadvantage.* The principal services of this nature are the *pumping of water into storage tanks and the charging of storage batteries.*

Windmill power equipment may be classified as follows :

1. *The multi-bladed turbine wheel.* This is the foremost type in use and its efficiency is about 10 per cent of the kinetic energy of the wind passing through it.
2. *The high-speed propeller type.*
3. The rotor.

The propeller and rotor types are *suitable for the generation of electrical energy,* as both of them possess the ability to start in very low winds. The *Propeller type is more likely to be used in small units* such as the driving of small battery charging generators, whereas the *rotor,* which is rarely seen, is more practical *for large installations,* even of several hundred kilowatts' capacity.

In india, the wind velocity along coastline has a range 10-16 kmph and a survey of wind power has revealed that wind power is capable of exploitation for pumping water from deep wells or for generating small amounts of electric energy.

Modern windmills are capable of working on velocities as low as 3-7 kmph while *maximum efficiency is attained at 10-12 kmph.*

A normal working life of 20 to 25 years is estimated for windmills.

The great advantage of this source of energy is that *no operator is needed and no maintenance and repairs are necessary for long intervals.*

Characteristics of wind power/energy. Some characteristics of wind energy are given below :

1. No fuel provision and transport are required in wind energy systems.
2. It is a renewable source of energy.
3. Wind power systems are non-polluting.
4. Wind power systems, upto a few kW, are less costly, but on a large scale, costs can be competitive with conventional electricity. Lower costs can be achieved by mass production.

Problems Associated with Wind Energy

1. Wind energy systems are noisy in operation.
2. Large areas are needed to install *wind farms* for electrical power generators.
3. Wind energy available is dilute and fluctuating in nature. Because of dilute form, conversion machines have to be necessarily large.
4. Wind energy needs storage means because of its irregularity.

1.2.5. Solar Energy

A lot of work to utilise solar energy for generation of steam has been done in some countries, particularly the U.S.S.R. and it is likely that this could be developed on commercial scale.

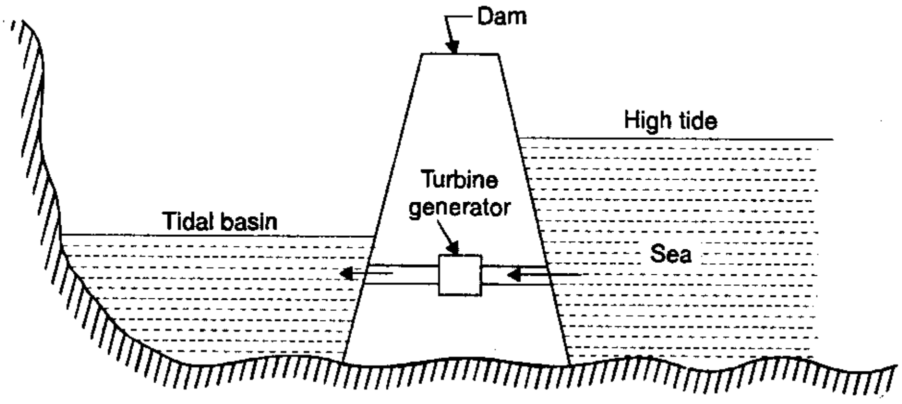
A serious fault of this source of energy is, of course, that it is effective only during the day, so that if a continuous output is needed, some large reservoir of energy, such as a storage battery or a heat accumulator tank, must be drawn upon at night. Also, the output is handicapped if there is cloudy weather. Nevertheless, there are some locations in the world where strong solar radiation is received very regularly, and where the sources of mineral fuel are either scanty or entirely lacking. Such locations offer more interest to the solar power plant builder than the more favoured regions of earth.

For developing solar energy two ways have been explored *viz.*, the glass lens and the reflector. These devices concentrate the solar rays to a focal point which is characterised by a high degree of heat which can be utilised to boil water and generate steam. The reflector is the better of the two methods due to the convenience with which it can be manufactured in different shapes and sizes. If an arrangement is provided to turn the reflector with the sun, so that the rays can constantly concentrate at the focal point, a continuous supply of heat is made available during the hours of the day. However, a great deal of practical research is still necessary before the solar energy can be commercially exploited at a cheaper rate.

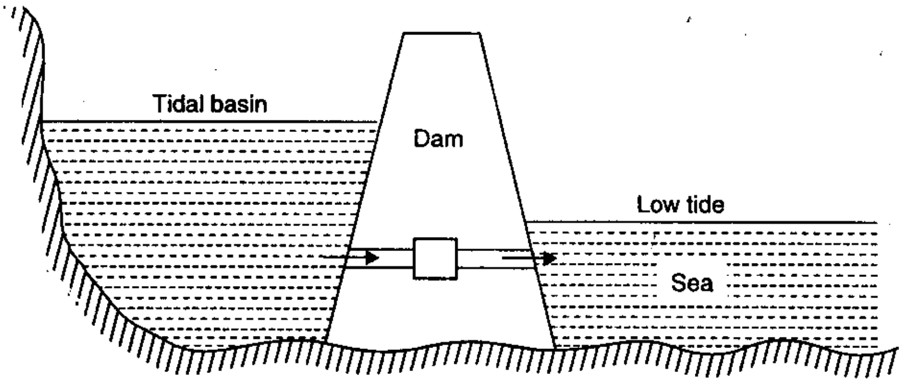
Conditions for utilisation of solar energy, in India, are favourable since for nearly six months of the year sunshine is uninterrupted during the day, while in the other six months cloudy weather and rain provide conditions suitable for water power. Thus, a coordination of solar energy with water power can provide a workable plan for most places in India.

1.2.6. Tidal Power

The rise and fall of tides offers a means for storing water at the rise and discharging the same at fall. Of course the head of water available under such cases is very low but with increased catchment area considerable amounts of power can be generated at a negligible cost.



(a)



(b)

Fig. 1.1. Generation of power by tides.

The use of tides for electric power generation is practical in a few favourably situated sites where the geography of an inlet of bay favours the construction of a large scale hydroelectric plant. To harness the tides, a dam would be built across the mouth of the bay in which large gates and low head hydraulic turbines would be installed. At the time of high tide the gates are opened and after storing water in the tidal basin the gates are closed. After the tide has received, there is a working hydraulic head between the basin water and open sea/ocean and the water is allowed to flow back to the sea through water turbines installed in the dam. With this type of arrangement, the generation of electric power is *not continuous*. However by using reversible water turbine the turbine can be run continuously as shown in Fig. 1.1.

1.2.7. Geothermal Energy

In many places on the earth natural steam escapes from surface vents. Such natural steam wells suggest the possibility of tapping terrestrial heat (or geothermal energy) in this form and using it for the development of power. Unfortunately, the locations where the steam-producing sub-strata seem to be fairly close to the surface are far removed from centres of civilization where the power could be usefully employed. Nevertheless, there are probably many places where, although no natural steam vent or hot springs are showing, deep drillings might tap a source of underground steam. The cost of such explorations and the great likelihood of an unsuccessful conclusion are not very conductive to exploitation of this source of energy.

There are two ways of electric power production from geothermal energy :

(*i*) Heat energy is transferred to a working fluid which operates the power cycle. This may be particularly useful at places of fresh volcanic activity where the molten interior mass of earth vents to the surface through fissures and substantially high temperatures, such as between 450 to 550°C can be found. By embedding coil of pipes and sending water through them steam can be raised.

(*ii*) The hot geothermal water and/or steam is used to operate the turbines directly. From the well-head the steam is transmitted by pipe lines upto 1 m in diameter over distances upto about 3 km to the power station. Water separators are usually employed to separate moisture and solid particles from steam.

Presently, only steam coming out of the ground is used to generate electricity, the hot water is discarded, because it contains as much as 30% dissolved salts and minerals, and these cause serious rust damage to the turbine. The water, however, contains more than $\frac{1}{3}$rd of the available thermal energy.

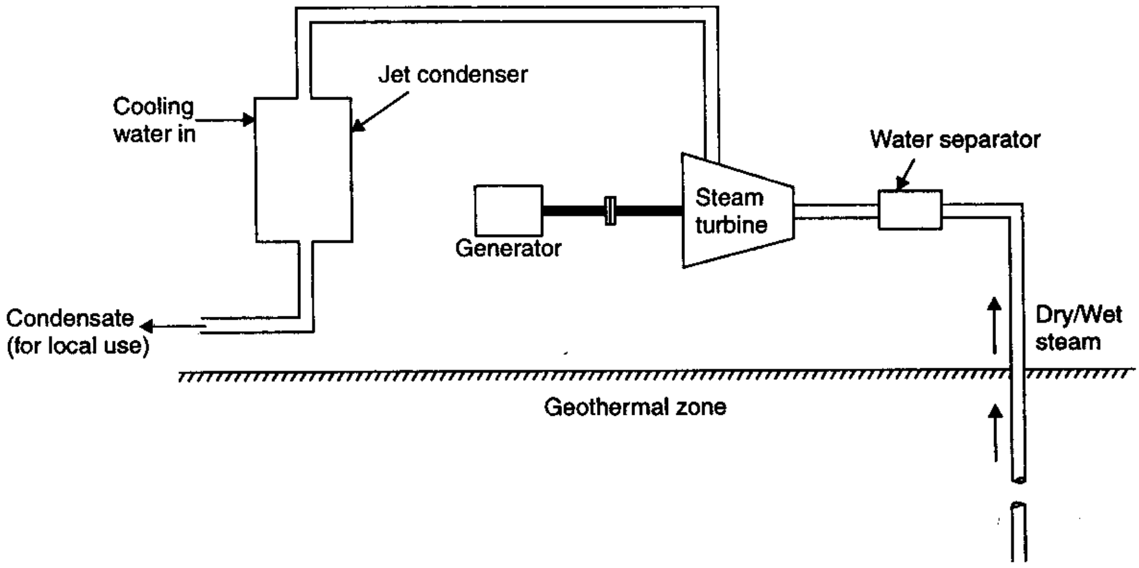


Fig. 1.2. Geothermal power plant.

1.2.8. Thermo-electric Power

According to *Seebeck effect*, when the two ends of a loop of two dissimilar metals are held at different temperatures, an electromotive force is developed and the current flows in loop. This method, by selection of suitable materials, can also be used for power generation. This method involves *low initial cost* and *negligible operating cost.*

1.3. PRINCIPAL TYPES OF POWER PLANTS

The principal types of power plants dealt in this book are :

1. Steam plants using coal, oil or nuclear fission.
2. Internal combustion engine plants.
3. Gas turbine plants.
4. Hydro-electric plants.

Steam plants using Coal, Oil or Nuclear Fission. The steam plants in which coal or oil is used comprise the *steam generating equipment* and the *primemovers.* In the boilers/steam generators steam is produced and then utilised to drive the *turbines/engines* which are *coupled to generators to get electricity.* The furnace may employ grate burning of *solid fuel, pulverised fuel* in burners or furnace oil in oil burners. Besides, the plant may contain several *heat saving devices* such as water heaters, economisers, air pre-heater etc. which effect an improvement in the overall efficiency of all boiler plants. The equipment for firing of fuel into the furnaces and also that for handling of fuel and ash are other important aspects of plant study besides such auxiliary equipment as needed for condensing exhaust steam, water treatment, water cooling, dust removal, draft control etc.

In nuclear stations heat is produced in a reactor which replaces the conventional boiler.

Internal Combustion Engine Plants. In these plants spark ignition (S.I.) or compression ignition (C.I.) engines are used as primemovers to drive electric generators. The *C.I. engines using diesel fuel are most commonly used* whereas the use of *S.I. engines are restricted to drive small portable electric generators.*

The diesel engine power plant may, besides the engine proper, include fuel handling, oil cooling, water cooling, starting and supercharging equipment.

Gas Turbine Plant. A gas turbine plant, working on a modified gas cycle, would contain besides the gas turbine, the *starting device, auxiliary lubrication, fuel control system, oil cooler, combustor, reheater, regenerator* etc. *A combination gas turbine plant employs an auxiliary steam plant obtaining its heat from gas turbine exhaust.* Such a combination plant has advantage of both gas and steam plants and some commercial plants are working on this system.

Hydro-electric Plants. This type of plant makes use of energy of water stored at an elevation and allowed to drop to a lower level. The electric generator is driven by a water turbine through which the water from the pond is made to work.

A hydro-electric plant comprises the *turbines, governing gear, coolant circulators* etc.

The *operation* of such a plant is usually *much simpler* than that of a steam, diesel or gas powered plant.

1.4. COMBUSTION OF FUELS

1.4.1. Combustion Chemistry

Under this heading we shall consider various important reactions concerning different elements which go to form the fuels and gases. Primarily there will be necessity to have some knowledge about the symbols and molecular weight of some important elements, compounds and gases. Table 1.1 given below satisfies the requirement.

Table 1.1

	Molecule		*Atom*	
	Symbol	*Molecule weight*	*Symbol*	*Molecular weight*
Hydrogen	H_2	2	H	1
Oxygen	O_2	32	O	16
Nitrogen	N_2	28	N	14
Carbon	C	12	C	12
Sulphur	S	32	S	32
Water	H_2O	18	—	—
Carbonmonoxide	CO	28	—	—
Carbondioxide	CO_2	44	—	—
Sulphurdioxide	SO_2	64	—	—
Marsh gas (Methane)	CH_4	16	—	—
Ethylene	C_2H_4	28	—	—
Ethane	C_2H_6	30	—	—

1. Carbon (complete combustion)

The complete combustion of carbon may be expressed by

$$C + O_2 = CO_2 \qquad ...(1.1)$$

or

$$1 \text{ kg (carbon)} + \frac{8}{3} \text{ kg (oxygen)} = \frac{11}{3} \text{ kg (carbondioxide)} \qquad ...(1.2)$$

Eqn. (1.1) indicates that one molecule of carbon combines with one molecule of oxygen to give one molecule of carbondioxide.

Eqn. (1.2) indicates that 1 kg of carbon needs 8/3 kg of oxygen for its complete combustion and produces 11/3 kg of carbondioxide.

2. Carbon (incomplete combustion)

Carbon combines with oxygen to form carbonmonoxide (and not carbondioxide as in the first case) according to the equation :

$$2C + O_2 = 2CO$$

$$2 \text{ mol} + 1 \text{ mol} = 2 \text{ mol}$$

or $$1 \text{ mol} + 1/2 \text{ mol} = 1 \text{ mol} \qquad ...(1.3)$$

$$24 + 32 = 56$$

or $$1 \text{ kg (carbon)} + 4/3 \text{ kg (oxygen)} = 7/3 \text{ kg (carbonmonoxide)} \qquad ...(1.4)$$

Eqn. (1.3) indicates that 1 molecule of carbon combines with $\frac{1}{2}$ molecule of oxygen to produce 1 molecule of carbonmonoxide.

Eqn. (1.4) represents that 1 kg of carbon requires 4/3 kg of oxygen to produce 7/3 kg of carbon monoxide.

3. Carbonmonoxide

It combines with oxygen to produce carbondioxide.

$$2CO + O_2 = 2CO_2$$

$$2 \text{ vol} + 1 \text{ vol} = 2 \text{ vol}$$

$$1 \text{ vol} + \tfrac{1}{2} \text{ vol} = 1 \text{ vol} \qquad ...(1.5)$$

$$56 + 32 = 88$$

or $$1 \text{ kg (carbon monoxide)} + 4/7 \text{ kg (oxygen)} = 11/7 \text{ kg (carbonmonoxide)} \qquad ...(1.6)$$

Eqn. (1.5) shows that one volume of carbonmonoxide combines with half volume of oxygen and produce one volume of carbonmonoxide gas.

Eqn. (1.6) indicates that 1 kg of carbonmonoxide reacts with 4/7 kg of oxygen to give 11/7 kg of carbondioxide.

4. Sulphur

Sulphur burns in the presence of oxygen to produce sulphurdioxide

$$S + O_2 = SO_2$$

$$1 \text{ mol} + 1 \text{ mol} = 1 \text{ mol} \qquad ...(1.7)$$

$$32 + 32 = 64$$

or $$1 \text{ kg (sulphur)} + 1 \text{ kg (oxygen)} = 2 \text{ kg (sulphurdioxide)} \qquad ...(1.8)$$

5. Hydrogen

Hydrogen burns with oxygen to give water as the product

$$2H_2 + O_2 = 2H_2O$$

$$1 \text{ vol} + \tfrac{1}{2} \text{ vol} = 1 \text{ vol} \qquad ...(1.9)$$

$$4 + 32 = 36$$

$$1 \text{ kg (hydrogen)} + 8 \text{ kg (oxygen)} = 9 \text{ kg (water)} \qquad ...(1.10)$$

6. Methane (Marsh gas)

When methane burns in the presence of oxygen the combustion products are carbon dioxide and water vapours,

$$CH_4 + 2O_2 = CO_2 + 2H_2O$$
$$1 \text{ vol} + 2 \text{ vol} = 1 \text{ vol} + 2 \text{ vol} \qquad ...(1.11)$$
$$16 + 64 = 44 + 36$$

or $$1 \text{ kg (methane)} + 4 \text{ kg (oxygen)} = 11/4 \text{ kg (carbondioxide)} + 9/4 \text{ kg (water)} \qquad ...(1.12)$$

7. Ethylene

On combustion with oxygen it also gives carbondioxide and water vapour as the products

$$C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$$
$$1 \text{ vol} + 3 \text{ vol} = 2 \text{ vol} + 2 \text{ vol} \qquad ...(1.13)$$
$$28 + 96 = 88 + 36$$

or $$1 \text{ kg (ethylene)} + 24/7 \text{ kg (oxygen)} = 22/7 \text{ kg (carbondioxide)} + 9/7 \text{ kg (water)} \qquad ...(1.14)$$

1.4.2. How to convert volumetric analysis to weight analysis ?

The conversion of volumetric analysis to weight analysis involves the following *steps* :

1. Multiply the volume of each constituent by its molecular weight.
2. Add all these weights and then divide each weight by the total of all and express it as percentage.

1.4.3. How to convert weight analysis to volumetric analysis ?

1. Divide the weight of each constituent by its molecular weight.
2. Add up these volumes and divide each volume by the total of all and express it as a percentage.

1.4.4. Weight of Air Required for Complete Combustion of Fuel

The exact amount of oxygen required, *theoretically*, for complete combustion of one kg of fuel can be determined from the analysis of the fuel as follows :

Firstly, the amount of oxygen required for each of the constituents of the fuel is calculated separately with the help of chemical equations.

Then these requirements are added and the total amount of oxygen required is obtained.

(If some oxygen is already present in the fuel then it must be *deducted* from the calculated amount of oxygen required for the combustion of the constituents).

The oxygen for the combustion of fuel has to be obtained from atmospheric air, which consists of oxygen, nitrogen, a small amount of carbondioxide and small traces of rare gases like neon, argon etc. But for all engineering calculations, composition of air is taken as follows :

$$\textit{By weight......} \begin{Bmatrix} \text{Nitrogen} = 77\% \\ \text{Oxygen} \;\; = 23\% \end{Bmatrix}$$

$$\textit{By volume .....} \begin{Bmatrix} \text{Nitrogen} = 77\% \\ \text{Oxygen} \;\; = 21\% \end{Bmatrix}$$

Thus the amount of oxygen required for the combustion of certain fuel is known, the amount of air necessary for the combustion of 1 kg of fuel can be determined.

"The theoretical or stoichiometric quantity of air is that quantity which is required for complete combustion of 1 kg of fuel without any oxygen appearing in the products of combustion."

1.4.5. Excess Air

For combustion of fuel we need air. As per theoretical basis there is a minimum amount of air which is required by the fuel to burn completely, *but always, air in excess is used because whole of air supplied for combustion purposes does not come in contact with the fuel completely and as such a* *portion of fuel may be left unburnt.* But if a large quantity of excess air is used it exercises *a cooling effect on combustion process which however can be avoided by preheating the air. The weight of excess air supplied can be determined from the weight of oxygen which is left unused.*

The amount of excess air supplied varies with the type of fuel and the firing conditions. It may approach a value of 100% but modern practice is to use 25% to 50% excess air.

1.4.6. Weight of Carbon in Flue Gases

The weight of carbon contained in one kg of flue or exhaust gas can be calculated from the amounts of CO_2 and CO contained in it.

In Eqn. (1.2) it was shown that 1 kg of carbon produces 11/3 kg of CO_2 when completely burnt. Hence 1 kg of CO_2 will contain 3/11 kg of carbon.

In Eqn. (1.4) it can be seen that 1 kg of carbon produces 7/3 kg of CO, hence 1 kg CO contains 3/7 kg of carbon.

Therefore, weight of carbon per kg of fuel

$$= \left(\frac{3}{11} CO_2 + \frac{3}{7} CO\right)$$

where CO_2 and CO are the quantities of carbondioxide and carbonmonoxide present in 1 kg of flue or exhaust gas.

1.4.7. Weight of Flue Gas per kg of Fuel Burnt

Due to supply of air, the weight of flue gas or exhaust gas is always more than that of fuel burnt. The actual weight of dry flue gases can be obtained by comparing the weight of carbon present in the flue gases with the weight of carbon in the fuel, since there is no loss of carbon during the combustion process. As the analysis of the exhaust gases is volumetric, so *this must first be reduced to weight analysis.*

Also, *total weight of carbon in one kg of flue gas is*

$$= \left(\frac{3}{11} CO_2 + \frac{3}{7} CO\right)$$

∴ *The weight of flue gas / kg of fuel burnt*

$$= \frac{\text{Weight of carbon in one kg of fuel}}{\text{Weight of carbon in one kg of flue gas}} .$$

WORKED EXAMPLES

Example 1.1. *The percentage composition of sample of liquid fuel by weight is, C = 84.8 percent, and H_2 = 15.2 percent. Calculate (i) the weight of air needed for the combustion of 1 kg of fuel ; (ii) the volumetric composition of the products of combustion if 15 percent excess air is supplied.*

Solution.

Element, wt (kg)	*O_2 used (kg)*	*Dry products (kg)*
C = 0.848	$0.848 \times \frac{8}{3} = 2.261$	$\frac{0.848 \times 11}{3}$ $= 3.109\ (CO_2)$
H_2 = 0.152	$0.152 \times 8 = 1.216$	
	Total O_2 = $\underline{3.477}$	

(*i*) *Minimum weight of air needed for combustion*

$$= \frac{3.477 \times 100}{23} = \mathbf{15.11\ kg.} \quad \textbf{(Ans.)}$$

Excess air supplied $= \frac{15.11 \times 15}{100} = 2.266$ kg

Wt. of oxygen in excess air $= \frac{2.266 \times 23}{100} = 0.521$ kg

Total air supplied for combustion

= Minimum air + Excess air = 15.11 + 2.266 = 17.376 kg

∴ Wt. of nitrogen (N_2) in flue gases

$$= \frac{17.376 \times 77}{100} = 13.38 \text{ kg.}$$

(*ii*) To get *volumetric composition of the products of combustion* let us use tabular method.

Name of gas	*Weight (x)*	*Molecular weight (y)*	*Proportional volume* $(z) = \frac{(x)}{(y)}$	*Percentage volume* $= \frac{(z)}{\Sigma(z)} \times 100$
CO_2	3.109	44	.0707	12.51 percent **Ans.**
O_2	0.521	32	.0163	2.89 percent **Ans.**
N_2	13.38	28	.4780	84.60 percent **Ans.**
			$\Sigma z = 0.5650$	

Example 1.2. *The percentage composition of a sample of coal is* $C = 90$; $H_2 = 3.5$; $O_2 = 3.0$; $N_2 = 1.0$; $S = 0.5$; *the remainder being ash. Estimate the minimum weight of air required for the combustion of 1 kg of this fuel and the composition of the dry products of combustion, by volume, if 50 percent excess air is supplied.*

Solution.

Element, wt (kg)	O_2 *reqd. (kg)*	*Dry products (kg)*
C = 0.9	$0.9 \times \frac{8}{3} = 2.400$	$0.9 \times \frac{11}{3} = 3.30\ (CO_2)$
$H_2 = 0.335$	$0.035 \times 8 = 0.280$	...
$O_2 = 0.03$	...	...
$N_2 = 0.01$	...	$0.01\ (N_2)$
S = 0.005	$0.005 \times \frac{1}{1} = 0.005$	$.005 \times \frac{2}{1} = .01\ (SO_2)$
	Total $O_2 = \overline{\underline{2.685}}$	

Weight of O_2 to be supplied

= Wt. of O_2 reqd. – Wt. of O_2 already present in the fuel

= 2.685 – 0.03 = **2.655 kg. (Ans.)**

Minimum weight of air required for combustion

$$= 2.655 \times \frac{100}{23} = \mathbf{11.54\ kg.} \quad \textbf{(Ans.)}$$

As 50 percent excess air is supplied, the total wt. of air supplied

$$= 11.53 \times 1.5 = 17.30 \text{ kg}$$

Wt. of nitrogen (N_2) present in the air

$$= \frac{17.30 \times 77}{100} = 13.32 \text{ kg}$$

Wt. of N_2 already in the fuel $= 0.01$ kg

Total wt. of N_2 $= 13.32 + .01 = 13.33$ kg

Wt. of free oxygen (O_2) due to excess air

$$= \frac{(11.54 \times 0.5) \times 23}{100} = 1.327 \text{ kg}$$

The percentage composition of dry flue gases can be found out as given below :

Name of gas	*Weight (x)*	*Molecular weight (y)*	*Proportional volume* $(z) = \frac{(x)}{(y)}$	*Percentage volume* $= \frac{(z)}{\Sigma (z)} \times 100$
CO_2	3.30	44	0.0750	**12.65 percent**
N_2	13.33	28	0.4761	**80.30 percent**
O_2	1.326	32	0.0414	**6.98 percent**
SO_2	0.01	64	0.000156	**0.0002 percent**
			$\Sigma z = 0.592656$	**Ans.**

Example 1.3. *Percentage volumetric analysis of a sample of flue gases of a coal fired boiler gave* $CO_2 = 10.4$; $CO = 0.2$; $O_2 = 7.8$ *and* $N_2 = 81.6$ *(by difference). Gravemetric percentage analysis of coal was* $C = 78$, $H_2 = 6$, $O_2 = 3$ *and incombustible = 13. Estimate :*

(*i*) *Weight of dry flue gases per kg of fuel.*

(*ii*) *Weight of excess air per kg of fuel.*

Solution.

Element, wt (kg)	*O_2 reqd. (kg)*	*Dry products (kg)*
C = 0.78	$0.78 \times \frac{8}{3} = 2.08$	$0.78 \times \frac{11}{3} = 2.86 \ (CO_2)$
$H_2 = 0.06$	$0.06 \times 8 = 0.48$	
$O_2 = 0.03$	Total $O_2 = \overline{\underline{2.56}}$	

Minimum wt. of air needed for combustion

$$= (2.56 - 0.03) \times \frac{100}{23} = 11 \text{ kg.}$$

(*i*) **Weight of dry flue gases per kg of fuel :**

To determine the wt. of flue gases per kg of fuel let us use tabular method to convert volumetric analysis to analysis by weight.

Name of gas	*Vol. per m^3 of flue gas (x)*	*Molecular weight weight (y)*	*Relative weight $z = x \times y$*	*Weight per kg of flue gas = $\frac{z}{\Sigma z}$*
CO_2	0.104	44	4.576	0.1525
CO	0.002	28	0.056	0.0019
N_2	0.816	28	22.848	0.7616
O_2	0.078	32	2.496	0.0832
			Σz = 29.976 (say 30)	

Amount of carbon present per kg of gases

= Amount of carbon in 0.1525 kg of CO_2

+ Amount of carbon present in 0.0019 kg of CO

$$= \frac{3}{11} \times 0.1525 + \frac{3}{7} \times 0.0019 = 0.0416 + 0.0008 = 0.0424 \text{ kg.}$$

Also carbon in the fuel = 0.78 kg.

∴ *Weight of dry flue gas per kg of fuel*

$$= \frac{\text{Weight of carbon in 1kg of fuel}}{\text{Weight of carbon in 1kg of flue gas}} = \frac{0.78}{0.0424} = \mathbf{18.4 \text{ kg.}} \quad \textbf{(Ans.)}$$

(*ii*) **Weight of excess air per kg of fuel :**

Weight of excess oxygen per kg of flue gas = $0.0832 - \frac{4}{7} \times 0.0019$

$$\begin{bmatrix} 2CO + O_2 = 2CO_2 \\ 56 + 32 = 88 \\ 1 \text{ kg} + \frac{4}{7} \text{ kg} = \frac{11}{7} \text{ kg} \end{bmatrix}$$

= 0.0832 – 0.0011 (allowing for unburnt carbonmonoxide)

= 0.0821 kg

Weight of excess O_2 per kg of fuel

= 18.4 × 0.0821 = 1.51 kg

Weight of excess air per kg of fuel

$$= \frac{1.51 \times 100}{23} = \mathbf{6.56 \text{ kg.}} \quad \textbf{(Ans.)}$$

Example 1.4. *A petrol sample was found to have 86 percent carbon and 14 percent hydrogen by weight. When used in an engine the air supply is 90 percent of that theoretically required for complete combustion. Assuming that all the hydrogen is burnt and that the carbon burns to carbonmonoxide and carbondioxide so that there is no free carbon left, calculate the percentage analysis of dry exhaust gases by volume.*

Solution.

Element, wt. (kg)	*O_2 needed (kg)*
C = 0.86	0.86 × 8/3 = 2.29
H_2 = 0.14	0.14 × 8 = 1.12
	Total O_2 needed = $\overline{\underline{3.41}}$

Minimum quantity of air for complete combustion = $\frac{3.41 \times 100}{23}$ = 14.83 kg/kg of fuel.

Amount of less air supplied = $\frac{14.83 \times 10}{100}$ = 1.483 kg/kg of fuel.

Amount of air saved by burning carbon to carbonmonoxide instead of carbondioxide

$$= \text{Oxygen saved} \times \frac{100}{23} = \left[\frac{8}{3}(CO_2) - \frac{4}{3}(CO)\right] \times \frac{100}{23}$$

= 5.8 kg/kg of carbon.

Hence $\frac{1.483}{5.8}$ = 0.256 kg of carbon burns to carbonmonoxide and as such 0.86 – 0.256 = 0.604 kg of carbon burns to carbondioxide.

∴ Carbondioxide (CO_2) formed = 0.604 × 11/3 = 2.215 kg

Carbonmonoxide (CO) formed = 0.256 × 7/3 = 0.597 kg

Nitrogen in the air supplied = 14.83 × 77/100 × 0.9 = 10.3 kg.

Volumetric analysis of dry exhaust gases

Dry products	*Weight (kg) (x)*	*Molecular weight (y)*	*Proportional volume* $(z) = \frac{(x)}{(y)}$	*Percentage volume* $= \frac{(z)}{(\Sigma z)} \times 100$
CO_2	2.215	44	$\frac{2.215}{44} = 0.050$	$\frac{0.050}{0.439} \times 100 =$ **11.39%**
CO	0.597	28	$\frac{0.597}{28} = 0.201$	$\frac{0.021}{0.441} \times 100 =$ **4.78%**
N_2	10.3	28	$\frac{10.3}{28} = 0.368$	$\frac{0.368}{0.441} \times 1 =$ **83.82%**
			$\Sigma z = 0.439$	**Ans.**

Example 1.5. *A boiler in a power station was fired with coal having an ultimate composition of C = 84%, H_2 = 4.5%, O_2 = 4.0% and the rest non-combustibles and calorific value of 33900 kJ/kg. The coal was burnt with 85% excess air and the mean temperature of the flue gases leaving the boiler was found to be 315.5 °C when the ambient temperature in the boiler bouse was 32.22 °C. Assuming air to be consisting of 23% by weight of O_2 and 77% by weight of N_2 and specific heat of flue gases as 1.005 kJ/kg K, determine :*

(i) kg air actually supplied per kg coal burnt.

(ii) kg flue gases per kg fuel burnt.

(iii) The percentage of the energy of the coal that is carried away by flue gases per kg of coal burnt.

Solution.

Element, wt. (kg)	*O_2 required (kg)*	*Wt. of products (kg)*
C = 0.84	$0.84 \times \frac{8}{3} = 2.24$	$0.84 \times \frac{11}{3} = 3.08\ (CO_2)$
H_2 = 0.045	$0.045 \times 8 = 0.36$	
	Total O_2 = $\overline{\underline{2.60}}$	

Weight of O_2 already present in the fuel = 0.04

Weight of O_2 needed for combustion = 2.60 – 0.04 = 2.56 kg

Air required for combustion = 2.56 × 100/23 = 11.13 kg.

(i) Actual quantity of air supplied = 1.85 × 11.13 = **20.6 kg. (Ans.)**

Weight of N_2 in the air required for combustion

= 11.13 × 0.77 = 8.57 kg.

Weight of N_2 in excess air = 11.13 × 0.77 × 0.85 = 7.285 kg.

∴ Total N_2 = 8.57 + 7.285 = 15.85 kg

O_2 in excess air = 11.13 × 0.85 × 0.23 = 2.18 kg.

(ii) Total wt. of flue gases = Wt. of CO_2 + Wt. of N_2 + Wt. of O_2 in excess air

= 3.08 + 15.85 + 2.18

= **21.11 kg. (Ans.)**

Heat taken away by flue gases = Weight of flue gases × specific heat × rise of temperature

= 21.11 × 1.005 × (315.5 – 32.22) = 6010 kJ

(iii) %age energy of coal taken away by flue gases per kg

$$= \frac{6010}{33900} \times 100 = \mathbf{17.73\%.}\ \textbf{(Ans.)}$$

Example 1.6. *The following is the ultimate analysis of a sample of petrol by weight :*

Carbon = 85 percent ; Hydrogen = 15 percent.

Calculate the ratio of air to petrol consumption by weight if the volumetric analysis of the dry exhaust gas is :

CO_2 = 11.5 percent ; CO = 1.2 percent, O_2 = 0.9 percent ; N_2 = 86 percent.

Also find percentage excess air.

Solution.

Name of gases	*Volume per m^3 of flue gas (x)*	*Molecular weight (y)*	*Relative weight $z = x \times y$*	*Weight per kg of flue gas = $\frac{z}{\Sigma z}$*
CO_2	0.115	44	5.06	0.1700
CO	0.012	28	0.336	0.0113
O_2	0.009	32	0.288	0.0096
N_2	0.86	28	24.08	0.8091
			Σz = 29.76	

∴ Weight of carbon per kg of flue gas

= Weight of carbon in 0.17 kg of CO_2 + weight of carbon in 0.0113 kg of CO

$$= \frac{3}{11} \times 0.17 + \frac{3}{7} \times 0.0113 = 0.0512 \text{ kg}$$

∴ Weight of dry flue gas per kg of fuel

$$= \frac{0.85}{0.0512} = 16.6 \text{ kg}$$

Vapour of combustion = 9 × 0.15 = 1.35 kg

Total weight of gas = 16.6 + 1.35 = 17.95 kg per kg of fuel.

∴ Air supplied $= (17.95 - 1) = 16.95$ kg/kg of fuel

∴ *Ratio of air to petrol* = **16.95 : 1. (Ans.)**

Stoichiometric air
$$= \left[\left(0.85 \times \frac{8}{3}\right) + (0.15 \times 8)\right] \times \frac{100}{23}$$
= 15.07 kg per kg of fuel

∴ Excess air $= 16.95 - 15.07 = 1.88$ kg

∴ *%age excess air* $= \frac{1.88}{15.07} \times 100 =$ **12.47%. (Ans.)**

Example 1.7. *A sample of fuel has the following percentage composition :*
Carbon = 86 percent, Hydrogen = 8 percent.
Sulphur = 3 percent, Oxygen = 2 percent, Ash = 1 percent.
For an air-fuel ratio of 12 : 1, calculate :
(*i*) *Mixture strength as a percentage rich or weak.*
(*ii*) *Volumetric analysis of the dry products of combustion.*

Solution.

Element, wt. (*kg*)	O_2 *reqd.* (*kg*)
C = 0.86	$0.86 \times \frac{8}{3} = 2.29$
$H_2 = 0.08$	$0.08 \times 8 = 0.64$
S = 0.03	$0.03 \times \frac{1}{1} = 0.03$
$O_2 = 0.02$	
	Total $O_2 = \underline{\overline{2.96}}$

Weight of oxygen to be supplied per kg of fuel
$$= 2.96 - 0.02 = 2.94 \text{ kg}$$

Weight of minimum air required for complete combustion
$$= \frac{2.94 \times 100}{23} = 12.78 \text{ kg}$$

Hence *"correct"* fuel air ratio $= \frac{1}{12.78} : 1.$

But actual ratio is $\frac{1}{12} : 1.$

(*i*) ∴ *Mixture strength* $= \frac{12.78}{12} \times 100 = 106.5\%.$

This shows that mixture is **6.5% rich. (Ans.)**

Deficient amount of air $= 12.78 - 12 = 0.78$ kg

Amount of air saved by burning 1 kg of C to CO instead of CO_2
$$= \text{oxygen saved} \times \frac{100}{23}$$
$$= \left[\frac{8}{3}(CO_2) - \frac{4}{3}(CO)\right] \times \frac{100}{23} = 5.8 \text{ kg}$$

Hence $\frac{0.78}{5.8} = 0.134$ kg of carbon burns to CO and as such $0.86 - 0.134 = 0.726$ kg of carbon burns to CO_2.

$\therefore$ CO formed $= 0.134 \times \frac{7}{3} = 0.313$ kg

CO_2 formed $= 0.726 \times \frac{11}{3} = 2.662$ kg

N_2 supplied $= 12 \times 0.77 = 9.24$ kg

SO_2 formed $= 0.03 \times 2 = 0.06$ kg.

(*ii*) The percentage composition of dry flue gases is given as below :

Dry products	*Weight (kg)x*	*Molecular weight y*	*Proportional volume* $z = \frac{x}{y}$	*Percentage volume* $= \frac{z}{\Sigma z} \times 100$
CO	0.313	28	0.0112	**2.78 percent. Ans.**
CO_2	2.662	44	0.0605	**15.03 percent. Ans.**
N_2	9.24	28	0.3300	**81.97 percent. Ans.**
SO_2	0.06	64	0.0009	**0.22 percent. Ans.**
			$\Sigma z = 0.4026$	

Example 1.8. *Find an expression for the weight of air supplied per kg of fuel when carbon content of the fuel and the volumetric analysis of the flue gas is known.*

Show that carbon burnt to CO per kg of fuel is $\frac{(CO)(C)}{CO + CO_2}$ *where CO and* CO_2 *represent percentage of carbonmonoxide and carbondioxide in dry flue gas by volume and C represents percentage by weight of carbon that is actually burnt.*

Solution. Let C = Percentage of carbon, by weight, in the fuel burnt,

CO_2 = Percentage by volume of carbondioxide in the dry flue gas,

CO = Percentage by volume of carbonmonoxide in the dry flue gas, and

N_2 = Percentage by volume of nitrogen in the dry flue gas.

Consider 100 m^3 of the gas.

The actual weight of CO_2 $= CO_2 \times a \times b \times 44$ in 100 m^3 of gas.

The actual weight of CO $= CO \times a \times b \times 28$ in 100 m^3 of gas.

The actual weight of N_2 $= N_2 \times a \times b \times 28$ in 100 m^3 of gas.

The molecular weight of CO = 28.

The molecular weight of N_2 = 28.

Total mass of carbon per 100 m^3 of flue gas

$$= [a \times b \times 44 \times CO_2] \times \frac{12}{44} + [a \times b \times 28 \times CO] \times \frac{12}{28}$$

The reactions producing CO_2 and CO are

$$\underset{12}{C} + \underset{32}{O_2} = \underset{44}{CO_2}$$

$\therefore$ C per kg of CO_2 $= \frac{12}{44}$

Also

$$\underset{24}{2C} + \underset{32}{O_2} = \underset{56}{2CO}$$

$\therefore$ C per kg of CO $= \frac{12}{28}$

All the carbon available in carbondioxide and carbonmonoxide comes from carbon present in the fuel. Also it can be safely assumed that all nitrogen comes from air (this analysis being not applicable to the fuels containing nitrogen).

Total carbon in the flue gas = $a \times b \times 12(CO_2 + CO)$ per kg of carbon in the flue gas

$$= \frac{a \times b \times 28 \times N_2}{a \times b \times 12(CO + CO_2)} \qquad ...(i)$$

For every kg of fuel supplied for combustion $\frac{C}{100}$ kg of carbon pass out of the flue, whence the weight of N_2 per kg of fuel is

$$\frac{28N_2}{12(CO + CO_2)} \times \frac{C}{100} \text{ kg.}$$

If all this nitrogen has come from the atmosphere, then air supplied per kg of fuel

$$= \frac{28 \times N_2 \times C \times 100}{12(CO + CO_2) \times 100 \times 77}$$

(since air contains 77% nitrogen by weight)

$$\therefore \text{ Air per kg of fuel } = \frac{N_2 \times C}{\frac{100 \times 77 \times 12}{28 \times 100}(CO + CO_2)} = \frac{\mathbf{N_2 \times C}}{\mathbf{33(CO + CO_2)}}. \quad \textbf{(Ans.)}$$

It has been shown above that the total carbon of the flue gas is given by $a \times b \times 12\,(CO_2 + CO)$ and that in CO by $a \times b \times 12$ CO, whence the proportion of carbon burnt to CO is given by

$$\frac{a \times b \times 12 \times CO}{a \times b \times 12(CO + CO_2)} = \frac{CO}{CO + CO_2}$$

and if C is the proportion of carbon actually burnt per kg of fuel, then *carbon burnt to CO per kg of fuel* = $\frac{(CO)(C)}{CO + CO_2}$ **(Ans.)**

Example 1.9. *The following results were obtained in a trial on a boiler fitted with economiser :*

	CO_2	CO	O_2	N_2
Analysis of gas entering the economiser	*8.3*	*0*	*11.4*	*80.3*
Analysis of gas leaving the economiser	*7.9*	*0*	*11.5*	*80.6*

(*i*) *Determine the air leakage into the economiser if the carbon content of the fuel is 80 percent.*

(*ii*) *Determine the reduction in temperature of the gas due to air leakage if atmospheric temperature is 20°C and flue gas temperature is 410°C. Ash collected from ash pan is 15 percent by weight of the fuel fired.*

Take : c_p *for air = 1.005 kJ/kg K and* c_p *for flue gas = 1.05 kJ/kg K.*

Solution. (*i*) Air supplied $= \frac{N_2 \times C}{33(CO_2 + CO)}$

Air supplied on the basis of conditions at entry to the economiser

$$= \frac{80.3 \times 80}{33(8.3 + 0)} = 23.45 \text{ kg}$$

Air supplied on the basis of conditions at exit

$$= \frac{80.6 \times 80}{33(7.9 + 0)} = 24.73 \text{ kg}$$

∴ *Air leakage* = 24.73 – 23.45 = **1.28 kg of air per kg of fuel. (Ans.)**

For each kg of fuel burnt, the ash collected is 15% *i.e.* 0.15 kg.

∴ Weight of fuel passing up the chimney = 1 – 0.15 = 0.85 kg.

∴ Total weight of products = Weight of air supplied per kg of fuel
+ weight of fuel passing through chimney per kg of fuel
= 23.45 + 0.85 = 24.3 kg.

Heat in flue gases per kg of coal
= Weight of flue gases × specific heat × temperature rise above 0°C
= 24.3 × 1.05 × (410 – 0) = 10461 kJ

Heat in leakage air = Weight of leakage air × specific heat
× temperature rise of air above 0°C
= 1.28 × 1.005 × (20 – 0) = 25.73 kJ

We can still consider, in the mixture, the gas and the air as separate and having their own specific heats, but sharing a common temperature *t*.

For heat balance :

$$(1.005 \times 1.28 + 24.3 \times 1.05)\,t = 10461 + 25.73$$

$$26.8t = 10486.73$$

$$\therefore \quad t = 391.3°\text{C}.$$

∴ *Fall in temperature as a result of the air leakage into the economiser*
= 410 – 391.3 = **18.7°C. (Ans.)**

Example 1.10. *The ultimate analysis of a sample of coal gave the following analysis by weight C = 65 percent,* H_2 *= 6 percent, S = 1.5 percent,* O_2 *= 18 percent. Find the calorific value of this coal by using Dulong's formula.*

Solution. Dulong's formula

$$\text{H.C.V.} = \frac{1}{100}\left[33800\,\text{C} + 144000\left(\text{H} - \frac{0}{8}\right) + 9270\,\text{S}\right]\text{kJ}$$

$$= \frac{1}{100}\left[33800 \times 65 + 144000\left(6 - \frac{18}{8}\right) + 9270 \times 1.5\right]$$

$$= \frac{1}{100}\,[2197000 + 540000 + 13905]$$

$$= \frac{1}{100}\,(2750905) = 27509 \text{ kJ/kg.}$$

Also the combustion of 1 kg of coal produces 0.06 × 9 = 0.54 kg of steam.

$$\text{L.C.V.} = (\text{H.C.V.} - 2465\, m_w)$$

$$= (27509 - 2465 \times 0.54)$$

$$= (27509 - 1331) = \mathbf{26178 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

Example 1.11. *Find the gross thermal efficiency of boiler in which the quantity of steam raised per hour was 2250 kg and the coal consumption was 225 kg per hour.*

From the analysis by weight the dried fuel was found to contain 87% carbon and 4% hydrogen. The feed water temperature was 61°C, the boiler gauge pressure 5 bar, and dryness of steam 0.95. The calorific value of carbon is 33700 kJ/kg and that of hydrogen, 144600 kJ/kg.

Solution. Boiler gauge pressure = 5 bar

∴ Boiler absolute pressure = 5 + 1 = 6 bar

Dryness fraction of steam, $x = 0.95$
Feed water temperature $= 61°C$
Quantity of steam produced $= 2250$ kg/h
Coal consumption $= 225$ kg/h
From steam tables, corresponding to 6 bar
$h_f = 670.4$ kJ/kg
$h_{fg} = 2085$ kJ/kg
Heat of 1 kg of the steam,
$H = h_f + xh_{fg} = 670.4 + 0.95 \times 2085$
$= 670.4 + 1980.75$
$= 2651.15$ kJ/kg
Heat supplied to steam per kg $= 2651.15$ – heat contained in 1 kg of feed water
$= 2651.15 - 1 \times 4.18 \times (61 - 0)$
$= 2396.2$ kJ/kg
Total heat supplied to steam per hour $= 2396.2 \times 2250$ kJ
Calorific value of coal $= 33700\ C + 144600\ H$
$= 33700 \times 0.87 + 144600 \times 0.04$
$= 29319 + 5784 = 35103$ kJ/kg
Total heat input $= 35103 \times 225$ kJ

$$\therefore \qquad \eta_{boiler} = \frac{2396.2 \times 2250}{35103 \times 225} = \mathbf{68.26\%}. \quad \textbf{(Ans.)}$$

Example 1.12. *A boiler generates 360 kg of steam per hour at a pressure of 8 bar absolute. The dryness fraction of the steam may be taken 0.95. Coal used has the following composition per kg :*

Carbon = *0.70*
Hydrogen = *0.05*
Oxygen = *0.10*
Sulphur = *0.02*
Ash = *0.13*

If the hourly rate of burning is 60 kg and the feed water temperature is 25°C, calculate the boiler efficiency.

Solution. From steam tables, corresponding to 8 bar pressure,
$h_f = 720.9$ kJ/kg
$h_{fg} = 2046.5$ kJ/kg
Quantity of heat supplied per kg of steam
$= (h_f + xh_{fg}) - 1 \times 4.18 \times (25 - 0)$
$= 720.9 + 0.95 \times 2046.5 - 4.18 \times 25$
$= 720.9 + 1944.2 - 104.5$
$= 2560.6$ kJ/kg
Heat supplied per hour $= 2560.6 \times 360$ kJ
Calorific value of coal $= 33800\ C + 144000\ (H - 0/8) + 9270\ S$

$$= 33800 \times 0.7 + 144000 \left(0.50 - \frac{0.1}{8}\right) + 9270 \times 0.02$$

$= 23660 + 5400 + 185.4 = 29245.4$

Total heat input $= 60 \times 29245.4$ kJ

$$\therefore \qquad \eta_{boiler} = \frac{2560.6 \times 360}{60 \times 29245.4} \times 100$$

$$= \mathbf{52.53\%}. \quad \textbf{(Ans.)}$$

Example 1.13. *A water gas at 21°C and 1.033 bar is burned with dry air supplied at 21°C and 1.033 bar. The fuel gas composition is CO_2 = 6%, N_2 = 5.5%, H_2 = 48%, O_2 = 0.5%, CH_4 = 2%, CO = 38%. The orsat analysis of the flue gas showed CO_2 = 15.5%, O_2 = 4.76% and CO = 0.2%. Find the percentage of excess air supplied for combustion and the volume of the flue gas at 232°C and 1.03 bar formed per m^3 of the fuel.*

Solution. Let x mole be changing from CO to CO_2.

Reactants mole/mole of fuel	*Reaction*	*O_2 required*	*Products*			
			CO_2	*CO*	*O_2*	*N_2*
$CO_2 = 0.06$		—	0.06			
$N_2 = 0.055$		—				0.055
$H_2 = 0.48$	$H_2 + \frac{1}{2} O_2 = H_2O$	0.24				
$O_2 = 0.005$		-0.005				With O_2 =
$CH_4 = 0.02$	$CH_4 + 2O_2 = CO_2$ $+ 2H_2O$	0.04	0.02			$O_2 \times \frac{0.79}{0.21}$
$CO = 0.38$	$CO + \frac{1}{2} O_2 = CO_2$	$x/2$	x	$0.38 - x$		
	Total	$0.275 + x/2$	$0.08 + x$	$0.38 - x$	$O_2 - 0.275 - x/2$	$3.762\ O_2 + 0.055$

Let oxygen supplied be O_2 mol.

$\therefore$ Oxygen in exhaust = $(O_2 - 0.275 - x/2)$ mol.

But flue gas analysis is :

$$CO_2 = 0.155,\ CO = 0.002,\ O_2 = 0.0476.$$

Comparing the values from table and flue gas analysis

$$\frac{\dfrac{1}{0.08 + x}}{\dfrac{1}{0.38 - x}} = \frac{\dfrac{1}{0.155}}{\dfrac{1}{0.002}}$$

$$\therefore \qquad x = 0.374$$

Similarly,
$$\frac{\dfrac{1}{0.08 + x}}{\dfrac{1}{O_2 - 0.275 - x/2}} = \frac{\dfrac{1}{0.155}}{\dfrac{1}{0.0476}}$$

$$\therefore \qquad O_2 = 0.6014$$

Products per mole of fuel, $CO_2 = 0.454$, $CO = 0.006$, $O_2 = 0.1394$, $N_2 = 2.3174$

Total products 2.9168 mol per mol of fuel.

Unburnt CO $= 0.38 - x = 0.38 - 0.374 = 0.006$ mol

O_2 required to burn unburnt CO to $CO_2 = 0.03$ mol

$\therefore$ Excess oxygen in exhaust $= 0.1394 - 0.003 = 0.1364$

Theoretical oxygen needed $= 0.275 + \frac{0.374}{2} = 0.462$

Excess air = excess oxygen

$$= \frac{0.1364}{0.462} = \mathbf{0.2952 \text{ or } 29.52\%.} \quad \textbf{(Ans.)}$$

The products are at 21°C and 76 cm of Hg, changing this volume to 232°C and 1.03 bar

$$= \frac{1.033 \times 2.9168}{(21+273)} = \frac{1.03 \times V}{(232+273)}$$

$\therefore$ **Volume, V = 5.025 mol. (Ans.)**

HIGHLIGHTS

1. Energy appears in many forms, but has one thing in common—energy is possessed of the ability to produce a dynamic, vital effect.
2. Sources of energy are :
   - (*i*) Fuels
   - (*ii*) Energy stored in water
   - (*iii*) Nuclear energy
   - (*iv*) Wind power
   - (*v*) Solar energy
   - (*vi*) Tidal power
   - (*vii*) Geothermal energy
   - (*viii*) Thermoelectric power.
3. A chemical fuel is a substance which releases heat energy on combustion. The principal element of each fuel are carbon and hydrogen. Though sulphur is a combustible element too but its presence in the fuel is considered to be undesirable.
4. The principal types of power plants are :
   - (*i*) Steam plants using coal, oil or nuclear fission
   - (*ii*) Internal combustion engine plants
   - (*iii*) Gas turbine plants
   - (*iv*) Hydro-electric plants.
5. The weight of excess air supplied can be determined from the weight of oxygen which is left unused.
6. The weight of carbon per kg of fuel $= \frac{3}{11} CO_2 + \frac{3}{7} CO$

   where CO_2 and CO are the quantities of carbondioxide and carbonmonoxide present in 1 kg of flue or exhaust gas.

   The weight of flue gas/kg of fuel burnt $= \dfrac{\text{Weight of carbon in one kg of fuel}}{\text{Weight of carbon in one kg of flue gas}}$.

THEORETICAL QUESTIONS

1. Enumerate sources of energy.
2. What is a chemical fuel ? How does it differ from a nuclear fuel ?
3. How are chemical fuels classified ?
4. Explain briefly the following solid fuels :

   Lignites and brown coals, bituminous coal and coke.
5. List the advantages of liquid fuels.
6. Describe the following gaseous fuels :

   Coal gas, Coke-oven gas, Blast furnace gas and Producer gas.
7. What are the advantages of gaseous fuels ?
8. State the factors which go in favour of nuclear energy.

9. Write short notes on :
   Tidal power, wind power and thermoelectric power.
10. Name the principal types of power plants.
11. Why do we feel the necessity of using excess air for burning fuels ?
12. How can the following be calculated ?
    (*i*) Weight of carbon in flue gases.
    (*ii*) Weight of flue gas per kg of fuel burnt.

UNSOLVED EXAMPLES

1. A fuel has the following composition by weight : Carbon = 86% ; hydrogen = 11.75% and oxygen = 2.25%. Calculate the theoretical air supply per kg of fuel, and the weight of products of combustion per kg of fuel. **[Ans.** 13.98 kg ; 4.21 kg]
2. The volumetric analysis of a fuel gas is : CO_2 = 14% ; CO = 1% ; O_2 = 5% and N_2 = 80%. Calculate the fuel gas composition by weight. **[Ans.** 20.24% ; 0.93% ; 5.25% ; 73.58%]
3. The ultimate analysis of a dry coal burnt in a boiler gauge C = 84% ; H_2 = 9% and incombustibles 7 % by weight. Determine the weight of dry flue gases per kg of coal burnt, if volumetric combustion of the flue gas is : CO_2 = 8.75%, CO = 2.25% ; O_2 = 8% and N_2 = 81%. **[Ans.** 18.92 kg]
4. During a trial in a boiler, the dry flue gas analysis by volume was obtained as CO_2 = 13%, CO = 0.3%, O_2 = 6%, N_2 = 80.7%. The coal analysis by weight was reported as C = 62.4%, H_2 = 4.2%, O_2 = 4.5%, moisture = 15% and ash = 13.9%. Estimate :
   (*a*) Theoretical air required to burn 1 kg of coal.
   (*b*) Weight of air actually supplied per kg of coal, and
   (*c*) The amount of excess air supplied per kg of coal burnt. **[Ans.** 8.5 kg ; 11.5 kg ; 3 kg]
5. A steam boiler uses pulverised coal in the furnace. The ultimate analysis of coal (by weight) as received is : C = 78% ; H_2 = 3% ; O_2 = 3% ; ash = 10% and moisture = 5%. Excess air supplied is 30%. Calculate the weight of air to be supplied and weight of gaseous product formed per kg of coal burnt.
   [Ans. 13 kg ; CO_2 = 2.86 kg ; H_2 = 0.27 kg ; excess O_2 = 0.69 kg and N_2 = 9.81 kg per kg of coal]
6. The percentage composition by mass of a crude oil is given as follows : C = 90%, H_2 = 3.3%, O_2 = 3%, N_2 = 0.8% ; S = 0.9% and remaining incombustible. If 50% excess air is supplied, find the percentages of dry exhaust gases found by volume. **[Ans.** CO_2 = 12.7% ; SO_2 = 0.05% ; O_2 = 7%, N_2 = 80.25%]
7. In a boiler trial, the analysis of the coal used is as follows : C = 20%, H_2 = 4.5%, O_2 = 7.5%, remainder-incombustible matter :
   The dry flue gas has the following composition by volume :
   CO_2 = 8.5%, CO = 1.2%, N_2 = 80.3%, O_2 = 10% determine :
   (*i*) Minimum weight of air required per kg of coal.
   (*ii*) Percentage excess air. **[Ans.** (*i*) 3.56 kg, (*ii*) 63.2%]
8. The ultimate analysis of a sample of petrol by weight is : Carbon = 0.835 ; hydrogen = 0.165. Calculate the ratio of air to petrol consumption by weight, if the volumetric analysis of the dry exhaust gas is : Carbondioxide = 12.1 ; carbon-monoxide = 1.1 ; oxygen = 0.8 ; nitrogen = 85.4 percent. Also find the percentage excess air. **[Ans.** 16.265 : 1 ; 5.6%]
9. The percentage composition of a sample of fuel was found to be C = 85%, H_2 = 9%, S = 3%, O_2 = 1.5%, Ash = 1.5.%. For an air-fuel ratio of 12 : 1, calculate (*i*) the mixture strength as a percentage rich or weak, (*ii*) the volumetric analysis of the dry products of combustion.
   [Ans. 8.83% rich ; CO_2 = 12.16%, CO = 2.54%, N_2 = 84.83%, SO_2 = 0.47%]
10. The ultimate analysis of fuel used in a boiler is given as follows : C = 87%, H_2 = 4%, O_2 = 3.5% and the remainder is ash. The volume of CO_2 in the exhaust gases noted by CO_2 recorder is 12.5%.
    Find the percentage excess air supplied per kg of fuel. **[Ans.** 51.95%]
11. The ultimate analysis of coal used in a boiler is given as follows :
    C = 82%, H_2 = 5.4%, O_2 = 7%, N_2 = 0.5%, S = 0.6%, moisture = 1% and the remainder is ash.

The volumetric analysis of the exhaust gases determined by Orsat apparatus is given as follows :
$CO_2 = 9.6\%$, $CO = 1.2\%$, $O_2 = 7.8\%$ and $N_2 = 81.4\%$.
From fundamental determine the percentage of excess air supplied to the furnace of the boiler.
[**Ans.** 57%]

12. The percentage composition of a sample of crude oil was found by analysis to be :
$C = 90\%$, $H_2 = 3.3\%$, $O_2 = 3\%$, $N_2 = 0.8\%$, $S = 0.9\%$, Ash = 2%
If 50% excess air is supplied, find the percentage composition of the dry flue gases by volume.
[**Ans.** $CO_2 = 12.69\%$, $SO_2 = 0.05\%$, $O_2 = 7\%$, $N_2 = 80.26\%$]

13. The volumetric analysis of exhaust gases of an oil engine is : $CO_2 = 15\%$, $CO = 2.2\%$, $O_2 = 1.6\%$, Nitrogen = 81.2%.
Convert this analysis into analysis by weight. [**Ans.** $CO_2 = 21.6\%$, $CO = 2.2\%$, $O_2 = 1.7\%$, $N_2 = 74.5\%$]

14. The dry exhaust gas from an oil engine had the following percentage composition by volume :
$CO_2 = 8.85\%$, $CO = 1.2\%$, $O_2 = 6.8\%$, $N_2 = 83.15\%$.
The fuel had the percentage composition by weight as follows : $C = 84\%$, $H_2 = 14\%$, $O_2 = 2\%$.
Determine the weight of air supplied per kg of fuel burnt. [**Ans.** 21.08 kg]

15. In a boiler trial, the fuel used has the following analysis by weight $C = 85\%$, $H_2 = 4\%$ and ash = 11%. If the flue gas analysis gave $CO_2 = 12\%$, $O_2 = 8\%$ and nitrogen = 80% by volume, find per kg of dry fuel :
(*i*) Weight of necessary air.
(*ii*) Weight of excess air. [**Ans.** 11.25 kg, 6.57 kg]

2

Power Plant Cycles

2.1. INTRODUCTION

A *thermal power station* works on the *basic principle that heat liberated by burning fuel is converted into mechanical work by means of a suitable working fluid. The mechanical work is converted into electric energy by the help of generators.*

A working fluid goes through a repetitive cyclic change and this cyclic change involving heat and work is known as the *thermodynamic cycle.* Thus a *thermodynamic cycle is a series of operations, involving a heat source, a heat receiver, a machine or utilizer between the source and the receiver and a working substance.*

In a *steam power station*, heat is released by burning fuel, this heat is taken up by water which works as the working fuel. Water is converted into steam as it receives heat in a boiler. The steam then expands in a turbine producing mechanical work which is then converted into electrical energy through a generator. The exhaust steam from the turbine is then condensed in a condenser and condensate thereafter pumped to the boiler where it again receives heat and the cycle is repeated.

2.2. CLASSIFICATION OF POWER PLANT CYCLES

Thermal power plants, in general, may work on vapour cycles or gas power cycles.

Vapour power cycles—Classification :

1. Rankine cycle
2. Reheat cycle
3. Regenerative cycle
4. Binary vapour cycle.

Gas power cycles—Classification :

1. Otto cycle
2. Diesel cycle
3. Dual combustion cycle
4. Gas turbine cycles
   (*i*) Open cycle
   (*ii*) Closed cycle.

2.3. CARNOT CYCLE

Fig. 2.1 shows a Carnot cycle on *T-s* and *p-V* diagrams. It consists of (*i*) two constant pressure operations (4-1) and (2-3) and (*ii*) two frictionless adiabatics (1-2) and (3-4). These operations are discussed below :

1. **Operation (4-1).** 1 kg of boiling water at temperature T_1 is heated to form wet steam of dryness fraction x_1. Thus heat is absorbed at constant temperature T_1 and pressure p_1 during this operation.

2. **Operation (1-2).** During this operation steam is expanded isentropically to temperature T_2 and pressure p_2. The point '2' represents the condition of steam after expansion.

3. **Operation (2-3).** During this operation heat is rejected at constant pressure p_2 and temperature T_2. As the steam is exhausted it becomes wetter and cooled from 2 to 3.

4. **Operation (3-4).** In this operation the wet steam at '3' is compressed isentropically till the steam regains its original state of temperature T_1 and pressure p_1. Thus cycle is completed.

Refer *T-s* diagram :

Heat supplied at constant temperature T_1 [operation (4-1)] = Area 4-1-*b*-*a* = $T_1 (s_1 - s_4)$ or $T_1 (s_2 - s_3)$.

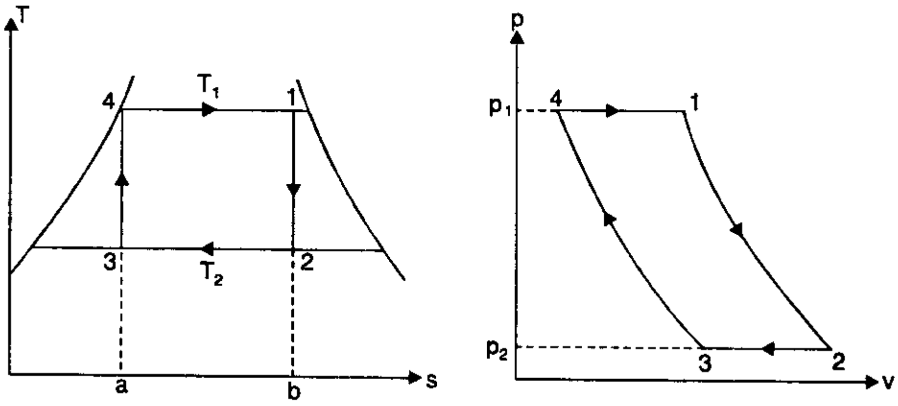


Fig. 2.1. Carnot cycle on *T-s* and *p-V* diagrams.

Heat rejected at constant temperature T_2 (operation 2-3) = Area 2-3-*a*-*b* = $T_2 (s_2 - s_3)$.

Since there is no exchange of heat during isentropic operations (1-2) and (3-4),

Net work done = Heat supplied – heat rejected

$$= T_1 (s_2 - s_3) - T_2 (s_2 - s_3)$$

$$= (T_1 - T_2)(s_2 - s_3).$$

$$\text{Carnot cycle } \eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{(T_1 - T_2)(s_2 - s_3)}{T_1 (s_2 - s_3)} = \frac{T_1 - T_2}{T_1} \qquad \ldots(2.1)$$

Limitations of Carnot cycle

Though Carnot cycle is simple (thermodynamically) and has the highest thermal efficiency for given values of T_1 and T_2, yet it is extremely difficult to operate in practice because of the following *reasons* :

1. It is difficult to compress a wet vapour isentropically to the saturated state as required by the process 3-4.

2. It is difficult to control the quality of the condensate coming out of the condenser so that the state '3' is exactly obtained.

3. The efficiency of the Carnot cycle is greatly affected by the temperature T_1 at which heat is transferred to the working fluid. Since the critical temperature for steam is only 374°C, therefore, if the cycle is to be operated in the *wet region*, the maximum possible temperature is severely limited.

4. The cycle is still more difficult to operate in practice with superheated steam due to the necessity of supplying the superheat at constant temperature instead of constant pressure (as it is customary).

In a practical cycle, limits of pressure and volume are far more easily realised than limits of temperature so that at present no practical engine operates on the Carnot cycle, although all modern cycles aspire to achieve it.

2.4. RANKINE CYCLE

Rankine cycle *is the theoretical cycle on which the steam turbine (or engine) works.*

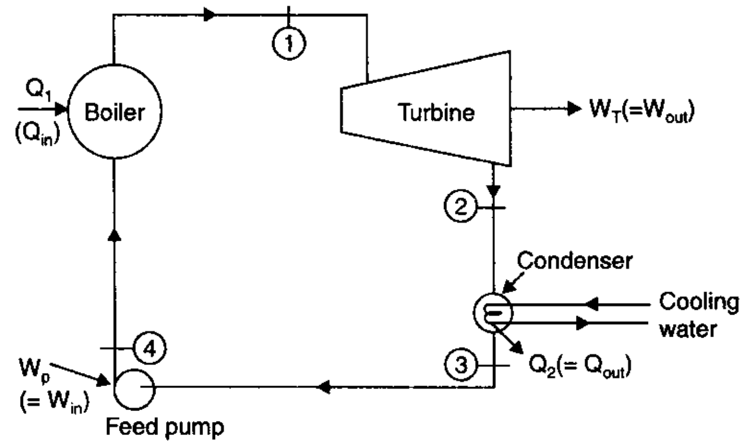
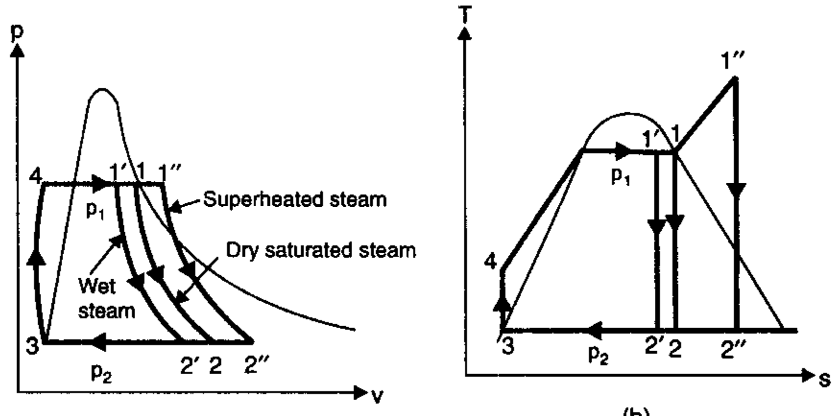
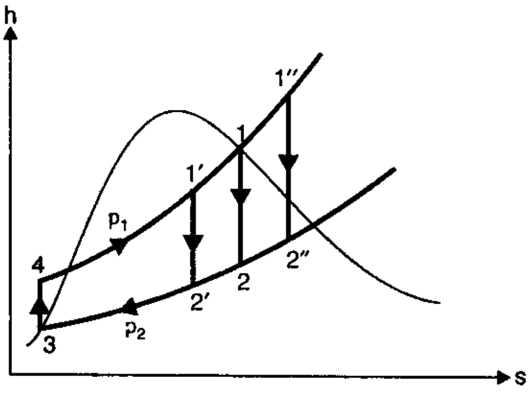


Fig. 2.2. Rankine cycle.



(a) (b)



(c)

Fig. 2.3. (*a*) *p-v* diagram ; (*b*) *T-s* diagram ; (*c*) *h-s* diagram for Rankine cycle.

The Rankine cycle is shown in Fig. 2.2. It consists of the following *processes* :

Process 1-2 : Reversible adiabatic expansion in the turbine (or steam engine).

Process 2-3 : Constant-pressure transfer of heat in the condenser.

Process 3-4 : Reversible adiabatic pumping process in the feed pump.

Process 4-1 : Constant-pressure transfer of heat in the boiler.

Fig. 2.3 shows the Rankine cycle on *p-v*, *T-s* and *h-s* diagrams (when the saturated steam enters the turbine, the steam can be wet or superheated also).

Consider 1 kg of fluid :

Applying *steady flow energy equation* (S.F.E.E.) to boiler, turbine, condenser and pump, we have :

(*i*) **For boiler** (as control volume), we get

$$h_{f_4} + Q_1 = h_1$$

$$\therefore \quad Q_1 = h_1 - h_{f_4} \qquad \text{...(2.2)}$$

(*ii*) **For turbine** (as control volume), we get

$$h_1 = W_T + h_2, \text{ where } W_T = \text{turbine work}$$

$$\therefore \quad W_T = h_1 - h_2 \qquad \text{...(2.3)}$$

(*iii*) **For condenser**, we get

$$h_2 = Q_2 + h_{f_3}$$

$$\therefore \quad Q_2 = h_2 - h_{f_3} \qquad \text{...(2.4)}$$

(*iv*) **For the feed pump**, we get

$$h_{f_3} + W_P = h_{f_4}, \quad \text{where, } W_P = \text{Pump work}$$

$$\therefore \quad W_P = h_{f_4} - h_{f_3}$$

Now, efficiency of Rankine cycle is given by

$$\eta_{\text{Rankine}} = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$= \frac{(h_1 - h_2) - (h_{f_4} - h_{f_3})}{(h_1 - h_{f_4})} \qquad ...(2.5)$$

The feed pump handles liquid water which is incompressible which means with the increase in pressure its density or specific volume undergoes a little change. Using general property relation for reversible adiabatic compression, we get

$$Tds = dh - vdp$$

$$\because \quad ds = 0$$

$$\therefore \quad dh = vdp$$

or $\Delta h = v\,\Delta p$ ...... (since change in specific volume is negligible)

or $h_{f_4} - h_{f_3} = v_3\,(p_1 - p_2)$

When p is in bar and v is in m³/kg, we have

$$h_{f_4} - h_{f_3} = v_3\,(p_1 - p_2) \times 10^5 \text{ J/kg}$$

The feed pump term $(h_{f_4} - h_{f_3})$ being a small quantity in comparison with turbine work, W_T, is usually neglected, *especially when the boiler pressures are low.*

Then, $$\eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_4}} \qquad ...[2.5\,(a)]$$

Comparison between Rankine cycle and Carnot cycle :

The following points are worth noting :

(*i*) Between the same temperature limits Rankine cycle provides a higher specific work output than a Carnot cycle, consequently Rankine cycle *requires a smaller steam flow rate resulting in smaller size plant for a given power output.* However, Rankine cycle calls for higher *rates of heat transfer in boiler and condenser.*

(*ii*) Since in Rankine cycle only part of the heat is supplied isothermally at constant higher temperature T_1, therefore, its *efficiency is lower* than that of Carnot cycle. The efficiency of the Rankine cycle will approach that of the Carnot cycle more nearly if the *superheat temperature rise is reduced.*

(*iii*) The advantage of using pump to feed liquid to the boiler instead to compressing a wet vapour is obvious that the *work for compression is very large compared to the pump.*

Fig. 2.4 shows the plots between efficiency and specific steam consumption against boiler pressure for Carnot and ideal Rankine cycles.

Effect of operating conditions on Rankine cycle efficiency :

The Rankine cycle efficiency can be *improved* by :

(*i*) Increasing the average temperature at which heat is supplied.

(*ii*) Decreasing/reducing the temperature at which heat is rejected.

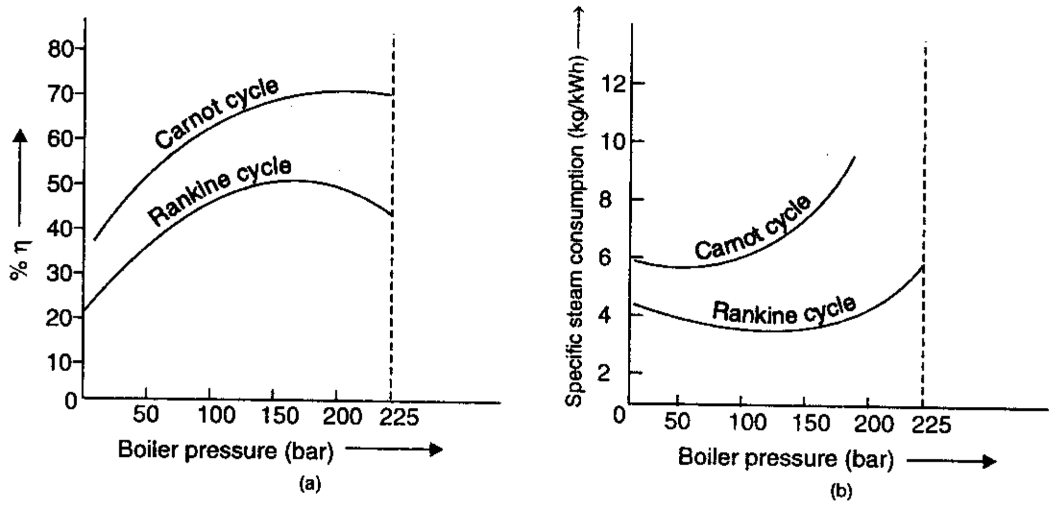


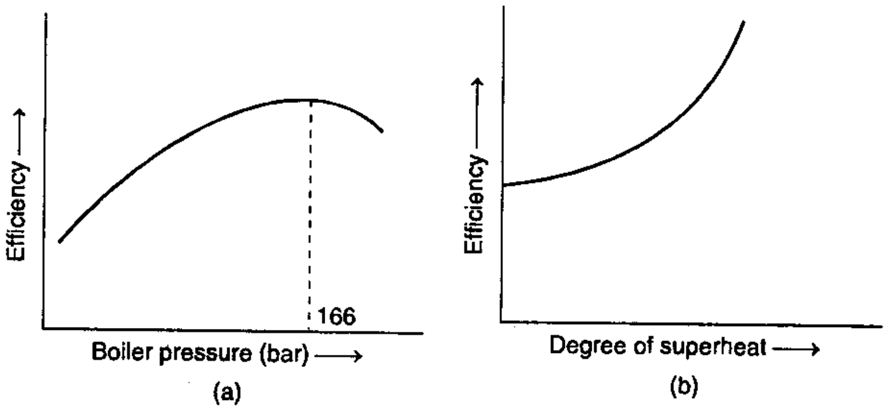
Fig. 2.4

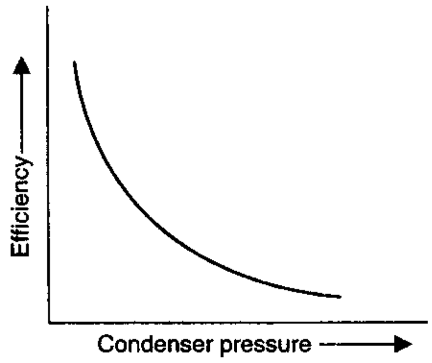
This can be achieved by making suitable changes in the conditions of steam generation or condensation, as discussed below :

1. **Increasing boiler pressure.** It has been observed that by increasing the boiler pressure (other factors remaining the same) the cycle tends to rise and reaches a maximum value at a boiler pressure of about 166 bar [Fig. 2.5 (*a*)].

2. **Superheating.** All other factors remaining the same, if the steam is superheated before allowing it to expand the *Rankine cycle efficiency may be increased* [Fig. 2.5 (*b*)]. The use of superheated steam also *ensures longer turbine blade life* because of the absence of erosion from high velocity water particles that are suspended in wet vapour.

3. **Reducing condenser pressure.** The thermal efficiency of the cycle can be amply improved by reducing the condenser pressure [Fig. 2.5 (*c*)] (hence by reducing the temperature at which heat is rejected), especially in high vacuums. But the increase in efficiency is obtained at the *increased cost of condensation apparatus.*





(c)

Fig. 2.5. Effect of operating conditions on the thermal efficiency of the Rankine cycle.

The thermal efficiency of the Rankine cycle is also *improved* by the following methods :

(*i*) *By regenerative feed heating.* (*ii*) *By reheating of steam.*

(*iii*) *By water extraction.* (*iv*) *By using binary-vapour.*

☞**Example 2.1.** *The following data refer to a simple steam power plant :*

S. No.	*Location*	*Pressure*	*Quality/temp.*	*Velocity*
1.	*Inlet to turbine*	*6 MPa (= 60 bar)*	*380°C*	—
2.	*Exit from turbine inlet to condenser*	*10 kPa (= 0.1 bar)*	*0.9*	*200 m/s*
3.	*Exit from condenser and inlet to pump*	*9 kPa (= 0.09 bar)*	*Saturated liquid*	—
4.	*Exit from pump and inlet to boiler*	*7 MPa (= 70 bar)*	—	—
5.	*Exit from boiler*	*6.5 MPa (= 65 bar)*	*400°C*	—
	Rate of steam flow = 10000 kg/h.			

Calculate :

(*i*) *Power output of the turbine.*

(*ii*) *Heat transfer per hour in the boiler and condenser separately.*

(*iii*) *Mass of cooling water circulated per hour in the condenser. Choose the inlet temperature of cooling water 20°C and 30°C at exit from the condenser.*

(*iv*) *Diameter of the pipe connecting turbine with condenser.*

Solution. Refer Fig. 2.6.

(*i*) **Power output of the turbine, P :**

At 60 bar, 380°C : From steam tables,

$$h_1 = 3043.0 \text{ (at 350°C)} + \frac{3177.2 - 3043.0}{(400 - 350)} \times 30 \text{ ... By interpolation}$$

$$= 3123.5 \text{ kJ/kg}$$

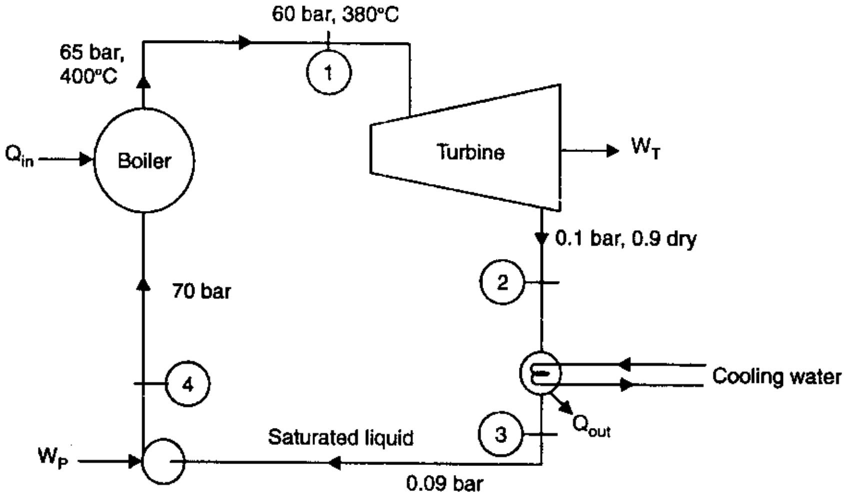


Fig. 2.6

At 0.1 bar :

$$h_{f_2} = 191.8 \text{ kJ/kg}, \; h_{fg_2} = 2392.8 \text{ kJ/kg (from steam tables)}$$

and $x_2 = 0.9$ (given)

$$\therefore \quad h_2 = h_{f_2} + x_2 h_{fg_2} = 191.8 + 0.9 \times 2392.8 = 2345.3 \text{ kJ/kg}$$

Power output of the turbine = $m_s (h_1 - h_2)$ kW,

[where m_s = Rate of steam flow in kg/s and h_1, h_2 = Enthalpy of steam in kJ/kg]

$$= \frac{10000}{3600} (3123.5 - 2345.3) = 2162 \text{ kW}$$

Hence power output of the turbine = **2162 kW. (Ans.)**

(*ii*) **Heat transfer per hour in the boiler and condenser :**

At 70 bar : $h_{f_4} = 1267.4$ kJ/kg

$$\textit{At 65 bar, 400°C} : h_a = \frac{3177.2\,(60 \text{ bar}) + 3158.1\,(70 \text{ bar})}{2} = 3167.6 \text{ kJ/kg}$$

......(By interpolation)

∴ Heat transfer per hour in the boiler,

$$Q_1 = 10000\,(h_a - h_{f_4}) \text{ kJ/h}$$
$$= 10000\,(3167.6 - 1267.4) = \mathbf{1.9 \times 10^7 \text{ kJ/h.}} \quad \textbf{(Ans.)}$$

At 0.09 bar : $h_{f_3} = 183.3$ kJ/kg

Heat transfer per hour in the condenser,

$$Q_2 = 10000\,(h_2 - h_{f_3})$$
$$= 10000\,(2345.3 - 183.3) = \mathbf{2.16 \times 10^7 \text{ kJ/h.}} \quad \textbf{(Ans.)}$$

(*iii*) **Mass of cooling water circulated per hour in the condenser, m_w :**

Heat lost by steam = Heat gained by the cooling water

$$Q_2 = m_w \times c_{pw} (t_2 - t_1)$$
$$2.16 \times 10^7 = m_w \times 4.18\,(30 - 20)$$

$$\therefore \qquad \mathbf{m_w} = \frac{2.16 \times 10^7}{4.18\,(30-20)} = \mathbf{1.116 \times 10^7\ kg/h.}\quad \textbf{(Ans.)}$$

(*iv*) Diameter of the pipe connecting turbine with condenser, d :

$$\frac{\pi}{4}\, d^2 \times C = m_s\, x_2\, v_{g_2} \qquad ...(i)$$

Here, d = Diameter of the pipe (m),

C = Velocity of steam = 200 m/s (given),

m_s = Mass of steam in kg/s

x_2 = Dryness fraction at '2',

v_{g_2} = Specific volume at pressure 0.1 bar (= 14.67 m^3/kg).

Substituting the various values in eqn. (*i*), we get

$$\frac{\pi}{4}\, d^2 \times 200 = \frac{10000}{3600} \times 0.9 \times 14.67$$

$$d = \left(\frac{10000 \times 0.9 \times 14.67 \times 4}{3600 \times \pi \times 200}\right)^{1/2} = 0.483 \text{ m or } 483 \text{ mm.}$$

Hence diameter of the pipe = **483 mm. (Ans.)**

Example 2.2. *A steam turbine receives steam at 15 bar and 350°C and exhausts to the condenser at 0.06 bar. Determine the thermal efficiency of the ideal Rankine cycle operating between these two limits.*

Solution. Pressure of steam at the entry to the steam turbine,

p_1 = 15 bar, 350°C

Condenser pressure, p_2 = 0.06 bar

Rankine efficiency :

From steam tables,

At 15 bar, 350°C : h = 3147.5 kJ/kg, s = 7.102 kJ/kg K

At 0.06 bar : h_f = 151.5 kJ/kg, h_{fg} = 2415.9 kJ/kg

s_f = 0.521 kJ/kg K, s_{fg} = 7.809 kJ/kg K

Since the steam in the turbine expands isentropically,

$$s_1 = s_2 = s_{f_2} + x_2 s_{fg_2}$$

$$7.102 = 0.521 + x_2 \times 7.809$$

$$\therefore \qquad x_2 = \frac{7.102 - 0.521}{7.809} = 0.843$$

$$h_1 = 3147.5 \text{ kJ/kg,}$$

$$h_2 = h_{f_2} + x_2 h_{fg_2} = 151.5 + 0.843 \times 2415.9 = 2188.1 \text{ kJ/kg}$$

$$\therefore \qquad \eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_2}} \text{ (neglecting pump work)}$$

$$= \frac{3147.5 - 2188.1}{3147.5 - 151.5} = \mathbf{0.32} \quad \textbf{or} \quad \mathbf{32\%.} \quad \textbf{(Ans.)}$$

Example 2.3. *In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam the net work and the cycle efficiency.*

Solution. Boiler pressure, $p_1 = 20$ bar (360°C)

Condenser pressure, $p_2 = 0.08$ bar

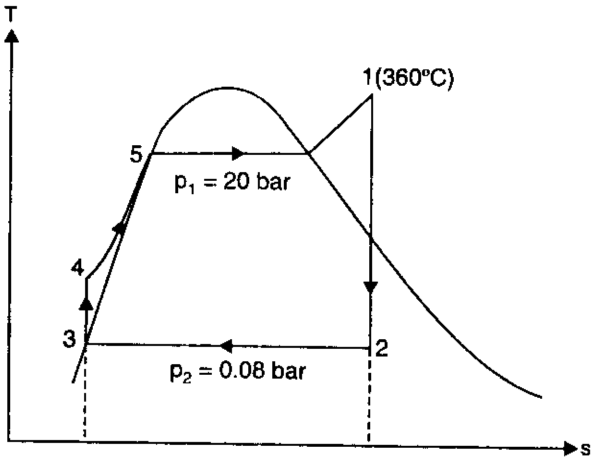


Fig. 2.7

From steam tables,

At 20 bar (p_1), 360°C : $h_1 = 3159.3$ kJ/kg

$s_1 = 6.9917$ kJ/kg-K

At 0.08 bar (p_2) : $h_3 = h_{f(p_2)} = 173.88$ kJ/kg,

$s_3 = s_{f(p_2)} = 0.5926$ kJ/kg K

$h_{fg(p_2)} = 2403.1$ kJ/kg, $s_{g(p_2)} = 8.2287$ kJ/kg K

$v_{f(p_2)} = 0.001008$ m³/kg $\therefore$ $s_{fg(p_2)} = 7.6361$ kJ/kg K

Now $s_1 = s_2$

$$6.9917 = s_{f(p_2)} + x_2 \, s_{fg(p_2)} = 0.5926 + x_2 \times 7.6361$$

$$\therefore \quad x_2 = \frac{6.9917 - 0.5926}{7.6361} = 0.838$$

$$\therefore \quad h_2 = h_{f(p_2)} + x_2 \, h_{fg(p_2)}$$

$$= 173.88 + 0.838 \times 2403.1 = 2187.68 \text{ kJ/kg.}$$

Net work, W_{net} :

$$W_{net} = W_{turbine} - W_{pump} \qquad \ldots(i)$$

$$W_{pump} = h_{f_4} - h_{f(p_2)} \; (= h_{f_3}) = v_{f(p_2)} \, (p_1 - p_2)$$

$$= 0.00108 \text{ (m}^3\text{/kg)} \times (20 - 0.08) \times 100 \text{ kN/m}^2$$

$$= 2.008 \text{ kJ/kg}$$

[and $h_{f_4} = 2.008 + h_{f(p_2)} = 2.008 + 173.88 = 175.89$ kJ/kg]

$$W_{turbine} = h_1 - h_2 = 3153.9 - 2187.68 = 971.62 \text{ kJ/kg}$$

$$\therefore \quad W_{net} = 971.62 - 2.008 = \mathbf{969.61 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

$$Q_1 = h_1 - h_{f_4} = 3159.3 - 175.89 = 2983.41 \text{ kJ/kg}$$

$$\therefore \quad \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{969.61}{2983.41} = \mathbf{0.325 \text{ or } 32.5\%.} \quad \textbf{(Ans.)}$$

Example 2.4. *A simple Rankine cycle works between pressures 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cycle efficiency, work ratio and specific steam consumption.*

Solution.

From steam tables,

At 28 bar : $h_1 = 2802$ kJ/kg, $s_1 = 6.2104$ kJ/kg K

At 0.06 bar : $h_{f_2} = h_{f_3} = 151.5$ kJ/kg, $h_{fg_2} = 2415.9$ kJ/kg,

$s_{f_2} = 0.521$ kJ/kg K, $s_{fg_2} = 7.809$ kJ/kg K

$v_f = 0.001$ m^3/kg

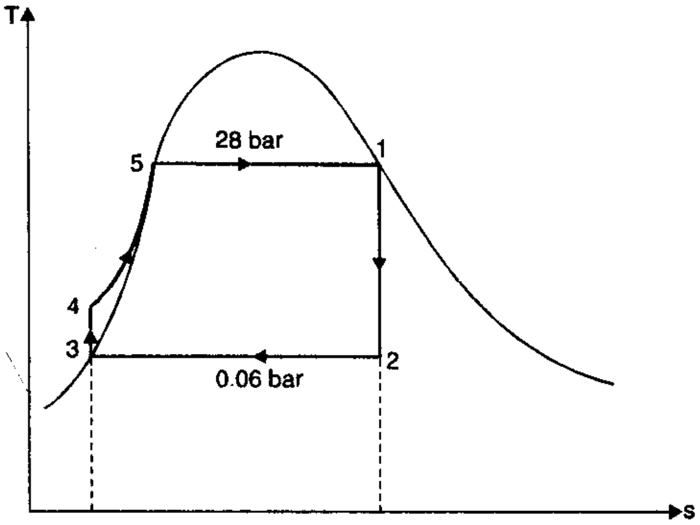


Fig. 2.8

Considering turbine *process 1-2*, we have :

$$s_1 = s_2$$

$$6.2104 = s_{f_2} + x_2\, s_{fg_2} = 0.521 + x_2 \times 7.809$$

$$\therefore \quad x_2 = \frac{6.2104 - 0.521}{7.809} = 0.728$$

$$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2} = 151.5 + 0.728 \times 2415.9 = 1910.27 \text{ kJ/kg}$$

$\therefore$ Turbine work, $W_{turbine} = h_1 - h_2 = 2802 - 1910.27 = 891.73$ kJ/kg

Pump work, $W_{pump} = h_{f_4} - h_{f_3} = v_f (p_1 - p_2)$

$$= \frac{0.001(28 - 0.06) \times 10^5}{1000} = 2.79 \text{ kJ/kg}$$

$$[\because \; h_{f_4} = h_{f_3} + 2.79 = 151.5 + 2.79 = 154.29 \text{ kJ/kg}]$$

∴ Net work, $W_{\text{net}} = W_{\text{turbine}} - W_{\text{pump}}$

$= 891.73 - 2.79 = 888.94$ kJ/kg

Cycle efficiency $= \dfrac{W_{\text{net}}}{Q_1} = \dfrac{888.94}{h_1 - h_{f_4}}$

$= \dfrac{888.94}{2802 - 154.29} = \mathbf{0.3357}$ **or 33.57%. (Ans.)**

Work ratio $= \dfrac{W_{\text{net}}}{W_{\text{turbine}}} = \dfrac{888.94}{891.73} = \mathbf{0.997}.$ **(Ans.)**

Specific steam consumption $= \dfrac{3600}{W_{\text{net}}} = \dfrac{3600}{888.94} = \mathbf{4.049}$ **kg/kWh. (Ans.)**

☞**Example 2.5.** *In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 35 bar and the exhaust pressure is 0.2 bar. Determine :*

(i) The pump work *(ii) The turbine work*

(iii) The Rankine efficiency *(iv) The condenser heat flow*

(v) The dryness at the end of expansion.

Assume flow rate of 9.5 kg/s.

Solution. Pressure and condition of steam, at inlet to the turbine,

$p_1 = 35$ bar, $x = 1$

Exhaust pressure, $p_2 = 0.2$ bar

Flow rate, $\dot{m} = 9.5$ kg/s

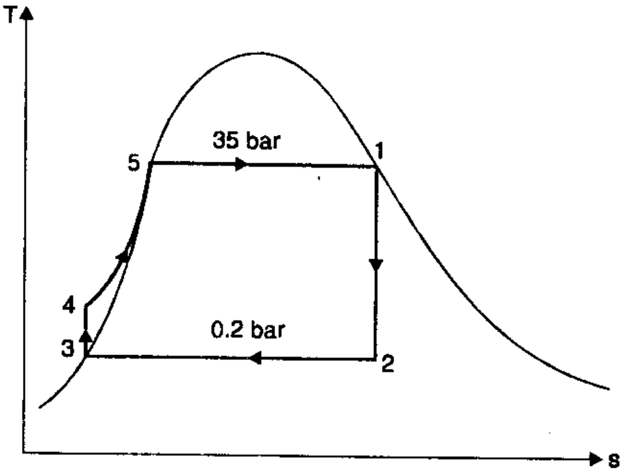


Fig. 2.9

From steam tables :

At 35 bar : $h_1 = h_{g_1} = 2802$ kJ/kg, $s_{g_1} = 6.1228$ kJ/kg K

At 0.26 bar : $h_f = 251.5$ kJ/kg, $h_{fg} = 2358.4$ kJ/kg,

$v_f = 0.001017$ m^3/kg, $s_f = 0.8321$ kJ/kg K, $s_{fg} = 7.0773$ kJ/kg K.

(*i*) **The pump work :**

Pump work $= (p_4 - p_3)\, v_f = (35 - 0.2) \times 10^5 \times 0.001017$ J or 3.54 kJ/kg

$$\left[\begin{array}{ll} \text{Also } h_{f_4} - h_{f_3} & = \text{Pump work} = 3.54 \text{ kJ / kg} \\ \therefore \quad h_{f_4} & = 251.5 + 3.54 = 255.04 \text{ kJ / kg} \end{array}\right]$$

Now power required to drive the pump

$$= 9.5 \times 3.54 \text{ kJ/s} \quad \text{or} \quad \mathbf{33.63\ kW.} \quad \textbf{(Ans.)}$$

(*ii*) **The turbine work :**

$$s_1 = s_2 = s_{f_2} + x_2 \times s_{fg_2}$$

$$6.1228 = 0.8321 + x_2 \times 7.0773$$

$$\therefore \quad x_2 = \frac{6.1228 - 0.8321}{7.0773} = 0.747$$

$$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2} = 251.5 + 0.747 \times 2358.4 = 2013 \text{ kJ/kg}$$

$$\therefore \quad \textit{Turbine work} = \dot{m}\,(h_1 - h_2) = 9.5\,(2302 - 2013) = \mathbf{7495.5\ kW.} \quad \textbf{(Ans.)}$$

It may be noted that pump work (33.63 kW) is very small as compared to the turbine work (7495.5 kW).

(*iii*) **The Rankine efficiency :**

$$\eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_2}} = \frac{2802 - 2013}{2802 - 251.5} = \frac{789}{2550.5} = \mathbf{0.3093\ or\ 30.93\%.} \quad \textbf{(Ans.)}$$

(*iv*) **The condenser heat flow :**

The condenser heat flow $= \dot{m}\,(h_2 - h_{f_3}) = 9.5\,(2013 - 251.5) = \mathbf{16734.25\ kW.}$ **(Ans.)**

(*v*) **The dryness at the end of expansion :**

The dryness at the end of expansion,

$$x_2 = \mathbf{0.747} \quad \textbf{or} \quad \mathbf{74.7\%.} \quad \textbf{(Ans.)}$$

2.5. MODIFIED RANKINE CYCLE

Figs. 2.10 and 2.11 show the modified Rankine cycle on *p-V* and *T-s* diagrams respectively. It will be noted that *p-V* diagram is very narrow at the toe *i.e.*, point '*n*' and the work obtained near toe is very small. In fact this work is too inadequate to overcome friction (due to reciprocating parts) even. Therefore, the adiabatic is terminated at '*s*' ; the pressure drop decreases suddenly whilst the volume remains constant. This operation is represented by the line *sq*. By this doing the stroke length is reduced ; in other words the cylinder dimensions reduce but at the expense of small loss of work (area '*sqn*') which, however, is negligibly small.

The work done during the modified Rankine cycle can be calculated in the following way :

Let p_1, v_1, u_1 and h_1 correspond to initial condition of steam at '*m*'.

p_2, v_2, u_2 and h_2 correspond to condition of steam at '*s*'.

p_3, h_3 correspond to condition of steam at '*n*'.

Work done during the cycle/kg of steam

$$= \text{area '}lmsqk\text{'}$$

$$= \text{area '}olmg\text{'} + \text{area '}gmsj\text{'} - \text{area '}okqj\text{'}$$

$$= p_1 v_1 + (u_1 - u_2) - p_3 v_2$$

Heat supplied $= h_1 - h_{f_3}$

∴ The modified Rankine efficiency

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{p_1v_1 + (u_1 - u_2) - p_3v_2}{h_1 - h_{f_3}} \quad \text{...(2.6)}$$

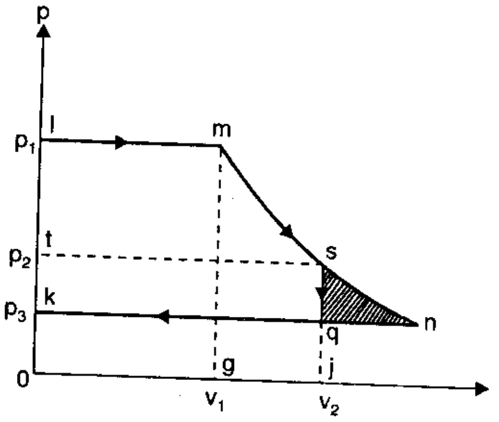


Fig. 2.10. *p-V* diagram of Modified Rankine cycle.

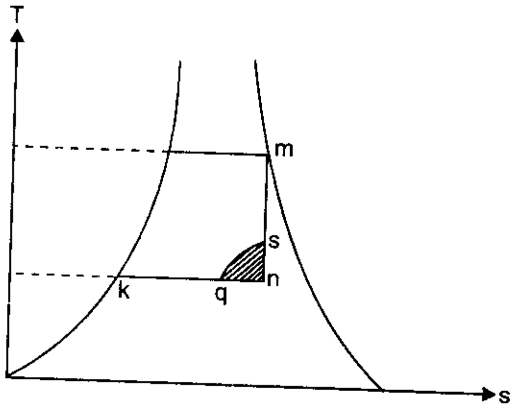


Fig. 2.11. *T-s* diagram of Modified Rankine cycle.

Alternative method for finding modified Rankine efficiency :

Work done during the cycle/kg of steam

= area '*lmsqk*'

= area '*lmst*' + area '*tsqk*'

$= (h_1 - h_2) + (p_2 - p_3)\, v_2$

Heat supplied $= h_1 - h_{f_3}$

Modified Rankine efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}}$

$$= \frac{(h_1 - h_2) + (p_2 - p_3)\, v_2}{h_1 - h_{f_3}} \quad \text{...(2.7)}$$

Note. Modified Rankine cycle is used for reciprocating steam engines because stroke length and hence cylinder size is reduced with the sacrifice of practically a quite negligible amount of work done.

Example 2.6. *Steam at a pressure of 15 bar and 300°C is delivered to the throttle of an engine. The steam expands to 2 bar when release occurs. The steam exhaust takes place at 1.1 bar. A performance test gave the result of the specific steam consumption of 12.8 kg/kWh and a mechanical efficiency of 80 percent. Determine :*

(*i*) *Ideal work or the modified Rankine engine work per kg.*

(*ii*) *Efficiency of the modified Rankine engine or ideal thermal efficiency.*

(*iii*) *The indicated and brake work per kg.*

(*iv*) *The brake thermal efficiency.*

(*v*) *The relative efficiency on the basis of indicated work and brake work.*

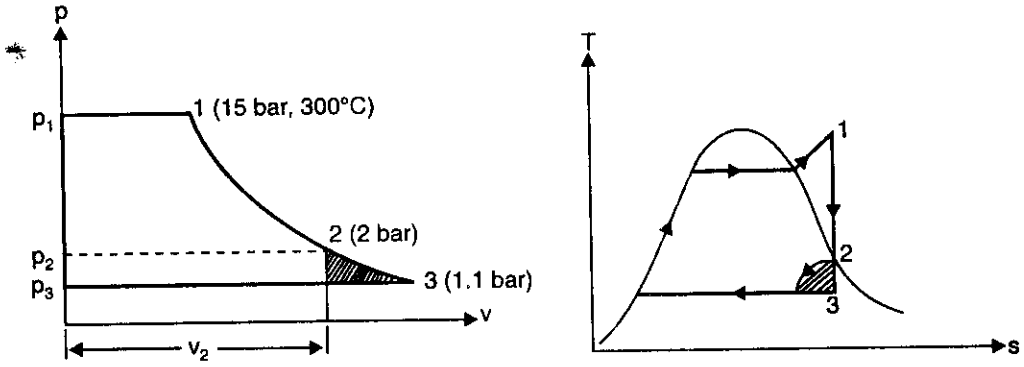
Solution. Fig. 2.12 shows the *p-v* and *T-s* diagrams for modified Rankine cycle.

From steam tables :

1. **At 15 bar, 300°C :** $h_1 = 3037.6$ kJ/kg, $v_1 = 0.169$ m^3/kg,
$s = 6.918$ kJ/kg K.

2. At 2 bar : $t_s = 120.2°\text{C}$, $h_f = 504.7$ kJ/kg, $h_{fg} = 2201.6$ kJ/kg,
$s_f = 1.5301$ kJ/kg K, $s_{fg} = 5.5967$ kJ/kg K,
$v_f = 0.00106$ m³/kg, $v_g = 0.885$ m³/kg.

3. At 1.1 bar : $t_s = 102.3°\text{C}$, $h_f = 428.8$ kJ/kg, $h_{fg} = 2250.8$ kJ/kg,
$s_f = 1.333$ kJ/kg K, $s_{fg} = 5.9947$ kJ/kg K,
$v_f = 0.001$ m³/kg, $v_g = 1.549$ m³/kg.



(a) (b)

Fig. 2.12

During isentropic expansion 1-2, we have

$$s_1 = s_2$$

$$6.918 = s_{f_2} + x_2\, s_{fg_2} = 1.5301 + x_2 \times 5.5967$$

$$\therefore \quad x_2 = \frac{6.918 - 1.5301}{5.5967} = 0.96.$$

Then $$h_2 = h_{f_2} + x_2 h_{fg_2} = 504.7 + 0.96 \times 2201.6 = 2618.2 \text{ kJ/kg}$$

$$v_2 = x_2\, v_{g_2} + (1 - x_2)\, v_{f_2}$$
$$= 0.96 \times 0.885 + (1 - 0.96) \times 0.00106 = 0.849 \text{ m}^3\text{/kg.}$$

(*i*) **Ideal work :**

Ideal work or modified Rankine engine work/kg,

$$W = (h_1 - h_2) + (p_2 - p_3)\, v_2$$
$$= (3037.6 - 2618.2) + (2 - 1.1) \times 10^5 \times 0.849/1000$$
$$= 419.4 + 76.41 = \mathbf{495.8 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

(*ii*) **Rankine engine efficiency :**

$$\eta_{\text{rankine}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{495.8}{(h_1 - h_{f_3})}$$

$$= \frac{495.8}{3037.6 - 428.8} = \mathbf{0.19 \text{ or } 19\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Indicated and brake work per kg :**

Indicated work/kg, $$W_{\text{indicated}} = \frac{\text{I.P.}}{\dot{m}} = \frac{1 \times 3600}{12.8} = \mathbf{281.25\ kJ/kg.}\ \ \textbf{(Ans.)}$$

Brake work/kg, $$W_{\text{brake}} = \frac{\text{B.P.}}{\dot{m}} = \frac{\eta_{\text{mech.}} \times \text{I.P.}}{\dot{m}} = \frac{0.8 \times 1 \times 3600}{12.8} = \mathbf{225\ kJ/kg.}\ \ \textbf{(Ans.)}$$

(*iv*) **Brake thermal efficiency :**

Brake thermal efficiency $$= \frac{W_{\text{brake}}}{h_1 - h_{f_3}} = \frac{225}{3037.6 - 428.8} = \mathbf{0.086\ or\ 8.6\%.}\ \ \textbf{(Ans.)}$$

(*v*) **Relative efficiency :**

Relative efficiency *on the basis of indicated work*

$$= \frac{\dfrac{W_{\text{indicated}}}{h_1 - h_{f_3}}}{\dfrac{W}{h_1 - h_{f_3}}} = \frac{W_{\text{indicated}}}{W} = \frac{281.25}{495.8} = \mathbf{0.567\ or\ 56.7\%.}\ \ \textbf{(Ans.)}$$

Relative efficiency *on the basis of brake work*

$$= \frac{\dfrac{W_{\text{brake}}}{(h_1 - h_{f_3})}}{\dfrac{W}{(h_1 - h_{f_3})}} = \frac{W_{\text{brake}}}{W} = \frac{225}{495.8} = \mathbf{0.4538\ or\ 45.38\%.}\ \ \textbf{(Ans.)}$$

Example 2.7. *Superheated steam at a pressure of 10 bar and 400°C is supplied to a steam engine. Adiabatic expansion takes place to release point at 0.9 bar and it exhausts into a condenser at 0.3 bar. Neglecting clearance determine for a steam flow rate of 1.5 kg/s :*

(*i*) *Quality of steam at the end of expansion and the end of constant volume operation.*

(*ii*) *Power developed.*

(*iii*) *Specific steam consumption.*

(*iv*) *Modified Rankine cycle efficiency.*

Solution. Fig. 2.13 shows the p-V and T-s diagrams for modified Rankine cycle.

From steam tables :

1. ***At 10 bar, 400°C :*** $h_1 = 3263.9$ kJ/kg, $v_1 = 0.307$ m³/kg, $s_1 = 7.465$ kJ/kg K
2. ***At 0.9 bar :*** $t_{s_2} = 96.7$°C, $h_{g_2} = 2670.9$ kJ/kg, $s_{g_2} = 7.3954$ kJ/kg K, $v_{g_2} = 1.869$ m³/kg
3. ***At 0.3 bar :*** $h_{f_3} = 289.3$ kJ/kg, $v_{g_3} = 5.229$ m³/kg

(*i*) **Quality of steam at the end of expansion,** T_{sup2} **:**

For isentropic expansion 1-2, we have

$$s_1 = s_2 = s_{g_2} + c_p \log_e \frac{T_{sup\,2}}{T_{s_2}}$$

$$7.465 = 7.3954 + 2.1 \log_e \frac{T_{sup2}}{(96.7 + 273)}$$

$$\left(\frac{7.465 - 7.3954}{2.1}\right) = \log_e \frac{T_{sup2}}{369.7} \quad \text{or} \quad \log_e \frac{T_{sup2}}{369.7} = 0.03314$$

$$\frac{T_{sup2}}{369.7} = 1.0337 \quad \text{or} \quad T_{sup2} = 382 \text{ K}$$

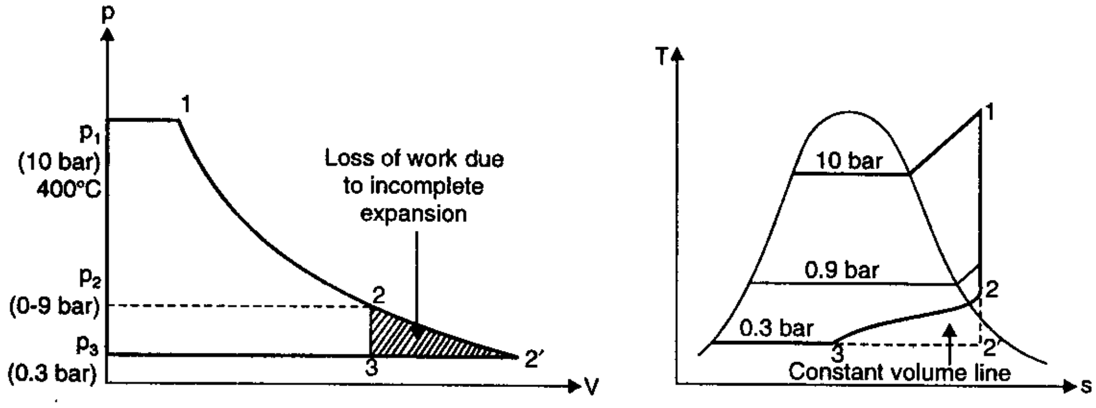


Fig. 2.13. *p-V* and *T-s* diagrams.

or $$t_{sup2} = 382 - 273 = \mathbf{109°C.} \quad \textbf{(Ans.)}$$

$\therefore$ $$h_2 = h_{g_2} + c_{ps}\,(T_{sup2} - T_{s_2})$$

$$= 2670.9 + 2.1\,(382 - 366.5) = 2703.4 \text{ kJ/kg.}$$

(*ii*) **Quality of steam at the end of constant volume operation, x_3 :**

For calculating v_2 using the relation

$$\frac{v_{g_2}}{T_{s_2}} = \frac{v_2}{T_{sup2}} \quad \text{(Approximately)}$$

$$\frac{1.869}{369.7} = \frac{v_2}{382}$$

or $$v_2 = \frac{1.869 \times 382}{369.7} = 1.931 \text{ m}^3\text{/kg}$$

Also $$v_2 = v_3 = x_3\, v_{g_3}$$

$$1.931 = x_3 \times 5.229 \quad \text{or} \quad \mathbf{x_3} = \frac{1.931}{5.229} = \mathbf{0.37.} \quad \textbf{(Ans.)}$$

(*iii*) **Power developed, P :**

Work done $$= (h_1 - h_2) + (p_2 - p_3)\, v_2$$

$$= (3263.9 - 2703.4) + \frac{(0.75 - 0.3) \times 10^5 \times 1.931}{1000}$$

$$= 560.5 + 86.9 = 647.4 \text{ kJ/kg}$$

$\therefore$ **Power developed** = Steam flow rate × work done (per kg)

$$= 1 \times 647.4 = \mathbf{647.4\ kW}. \quad \textbf{(Ans.)}$$

(*iv*) **Specific steam consumption, ssc :**

$$\text{ssc} = \frac{3600}{\text{Power}} = \frac{1 \times 3600}{647.4} = \mathbf{5.56\ kg/kWh}. \quad \textbf{(Ans.)}$$

(*v*) **Modified Rankine cycle efficiency, η_{mR} :**

$$\eta_{mR} = \frac{(h_1 - h_2) + (p_2 - p_3)\,v_2}{h_1 - h_{f_3}}$$

$$= \frac{647.4}{3263.9 - 289.3} = \mathbf{0.217\ or\ 21.7\%}. \quad \textbf{(Ans.)}$$

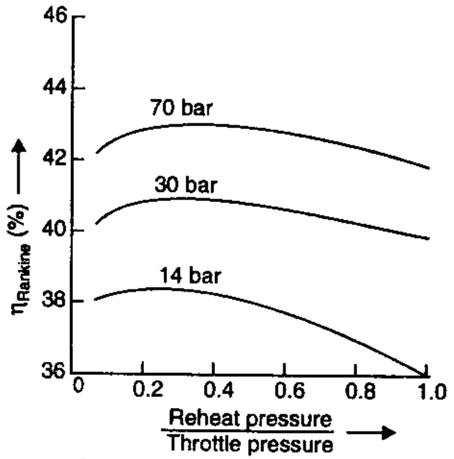
2.6. REHEAT CYCLE

For attaining greater thermal efficiencies when the initial pressure of steam was raised beyond 42 bar it was found that resulting condition of steam after, expansion was increasingly wetter and exceeded the safe limit of 12 per cent condensation. It, therefore, became necessary to *reheat* the steam after part of expansion was over so that the resulting condition after complete expansion fell within the region of permissible wetness.

The reheating or resuperheating of steam is now universally used when high pressure and temperature steam conditions such as 100 to 250 bar and 500°C to 600°C are employed for throttle. For plants of *still higher pressures and temperatures, a double reheating may be used.*

In actual practice reheat *improves* the cycle efficiency by about 5% for a 85/15 bar cycle. A *second reheat* will give a *much less gain* while the initial cost involved would be so high as to prohibit use of two stage reheat except in case of very high initial throttle conditions. The cost of reheat equipment consisting of boiler, piping and controls may be 5% to 10% more than of the conventional boilers and this additional expenditure is justified only if gain in thermal efficiency is sufficient to promise a return of this investment. *Usually a plant with a base load capacity of 50000 kW and initial steam pressure of 42 bar would economically justify the extra cost of reheating.*

The improvement in thermal efficiency due to reheat is greatly dependent upon the *reheat pressure* with respect to the original pressure of steam.



Condenser pressure : 12.7 mm Hg
Temperature of throttle & heat : 427°C

Fig. 2.14. Effect of reheat pressure selection on cycle efficiency.

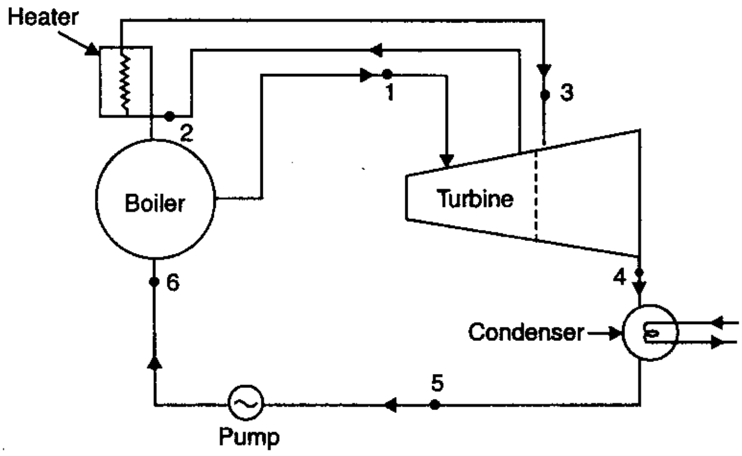
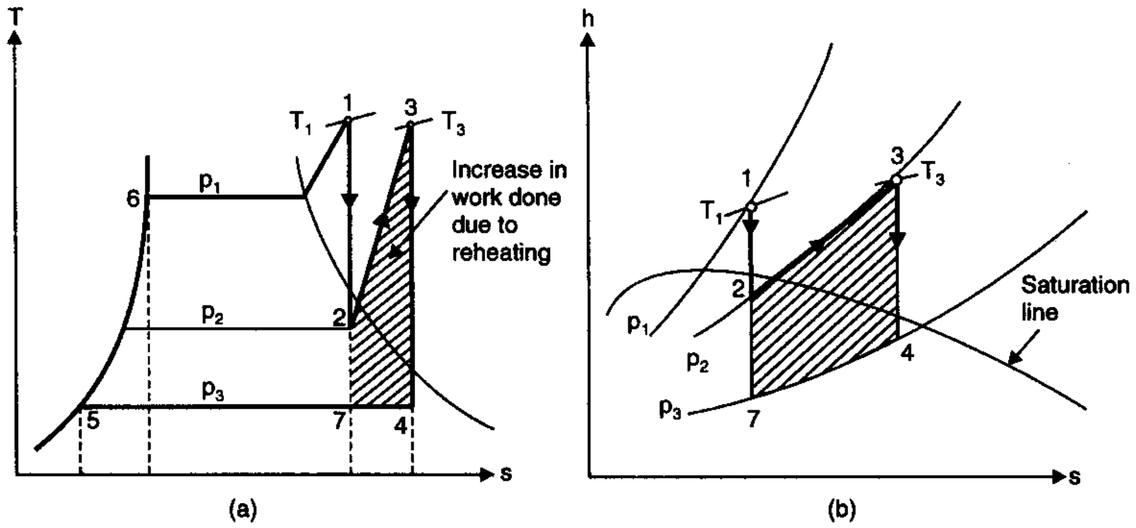


Fig. 2.15. Reheat cycle.

Fig. 2.15 shows a schematic diagram of a theoretical single stage reheat cycle. The corresponding representation of ideal reheating process on *T-s* and *H-s* chart are shown in Fig. 2.16 (*a*) and (*b*).

Refer Fig. 2.16, (*a*). 5-1 shows the formation of steam in the boiler. The steam as at state point 1 (*i.e.*, pressure p_1 and temperature T_1) enters the turbine and expands isentropically to a certain pressure p_2 and temperature T_2. From this state point 2 the whole of steam is drawn out of the turbine and is reheated in a reheater to a temperature T_3. (Although there is an *optimum pressure* at which the steam should be removed for reheating, if the highest return is to be obtained, yet, for simplicity, the whole steam is removed from the high pressure exhaust, where the pressure is about one-fifth of boiler pressure, and after undergoing a 10% pressure drop, in circulating through the heater, it is returned to intermediate pressure or low pressure turbine). This reheated steam is then readmitted to the turbine where it is expanded to condenser pressure isentropically.



(a) (b)

Fig. 2.16. Ideal reheating process on *T-s* and *h-s* chart.

Thermal efficiency with *'Reheating'* (neglecting pump work) :

Heat supplied $= (h_1 - h_{f_4}) + (h_3 - h_2)$

Heat rejected $= h_4 - h_{f_4}$

Work done by the turbine = Heat supplied – Heat rejected

$$= (h_1 - h_{f_4}) + (h_3 - h_2) - (h_4 - h_{f_4})$$

$$= (h_1 - h_2) + (h_3 - h_4)$$

Thus, theoretical thermal efficiency of reheat cycle is

$$\eta_{\text{thermal}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)} \quad ...(2.8)$$

If pump work, $W_p = \frac{v_f\,(p_1 - p_b)}{1000}$ kJ/kg is considered, the thermal efficiency is given by :

$$\eta_{\text{thermal}} = \frac{[(h_1 - h_2) + (h_3 - h_4)] - W_p}{[(h_1 - h_{f_4}) + (h_3 - h_2)] - W_p} \quad ...(2.9)$$

W_p is usually small and *neglected.*

Thermal efficiency *without reheating* is

$$\eta_{\text{thermal}} = \frac{h_1 - h_7}{h_1 - h_{f_4}} \qquad (\because \; h_{f_4} = h_{f_7}) \quad ...(2.10)$$

Note 1. The reheater may be incorporated in the walls of the main boiler ; it may be a separately fired superheater or it may be heated by a coil carrying high-pressure superheated steam, this system being analogous to a steam jacket.

2. Reheating should be done at *'optimum pressure'* because if the steam is reheated early in its expansion then the additional *quantity of heat supplied will be small* and thus thermal efficiency gain will be small ; and if the reheating is done at a fairly low pressure, then, although a large amount of additional heat is supplied, the steam will have a high degree of superheat (as is clear from Mollier diagram), thus a large proportion of the heat supplied in the reheating process *will be thrown to waste in the condenser.*

Advantages of 'Reheating' :

1. There is an increased output of the turbine.
2. Erosion and corrosion problems in the steam turbine are eliminated/avoided.
3. There is an improvement in the thermal efficiency of the turbines.
4. Final dryness fraction of steam is improved.
5. There is an increase in the nozzle and blade efficiencies.

Disadvantages :

1. Reheating requires more maintenance.
2. The increase in thermal efficiency is not appreciable in comparison to the expenditure incurred in reheating.

Superheating of Steam :

The primary object of superheating steam and supplying it to the primemovers is *to avoid too much wetness at the end of expansion.* Use of inadequate degree of superheat in *steam engines* would *cause greater condensation* in the engine cylinder ; while in case of *turbines* the moisture content of steam would result in *undue blade erosion.* The *maximum wetness in the final condition of steam that may be tolerated without any appreciable harm to the turbine blades is about 12 percent.* Broadly each *1 percent* of moisture in steam reduces the efficiency of that part of the turbine in which wet steam passes by 1 percent to 1.5 percent and in engines about 2 percent.

Advantages of Superheated steam :

1. Superheating reduces the initial condensation losses in steam engines.

2. Use of superheated steam results in *improving the plant efficiency* by effecting a *saving in cost of fuel*. This saving may be of the order of 6% to 7% due to first 38°C of superheat and 4 to 5% for next 38°C and so on. This saving results due to the fact that the heat content and consequently the capacity to do work in superheated steam is increased and the quantity of steam required for a given output of power is reduced. Although additional heat has to be added in the boiler there is reduction in the work to be done by the feed pump, the condenser pump and other accessories due to reduction in quantity of steam used. It is estimated that the *quantity of steam may be reduced by 10% to 15% for first 38°C of superheat and some what less for the next 38°C of superheat in the case of condensing turbines.*

3. When a superheater is used in a boiler it helps in *reducing the stack temperatures* by extracting heat from the flue gases before these are passed out of chimney.

☞**Example 2.8.** *Steam at a pressure of 15 bar abs. and 250°C is expanded through a turbine at first to a pressure of 4 bar abs. It is then reheated at constant pressure to the initial temperature of 250°C and is finally expanded to 0.1 bar abs. Using Mollier chart, estimate the work done per kg of steam flowing through the turbine and amount of heat supplied during the process of reheat. Compare the work output when the expansion is direct from 15 bar abs. to 0.1 bar abs. without any reheat. Assume all expansion processes to be isentropic.*

Solution. Refer Fig. 2.17.

Pressure, $p_1 = 15$ bar ;

$p_2 = 4$ bar ;

$p_4 = 0.1$ bar.

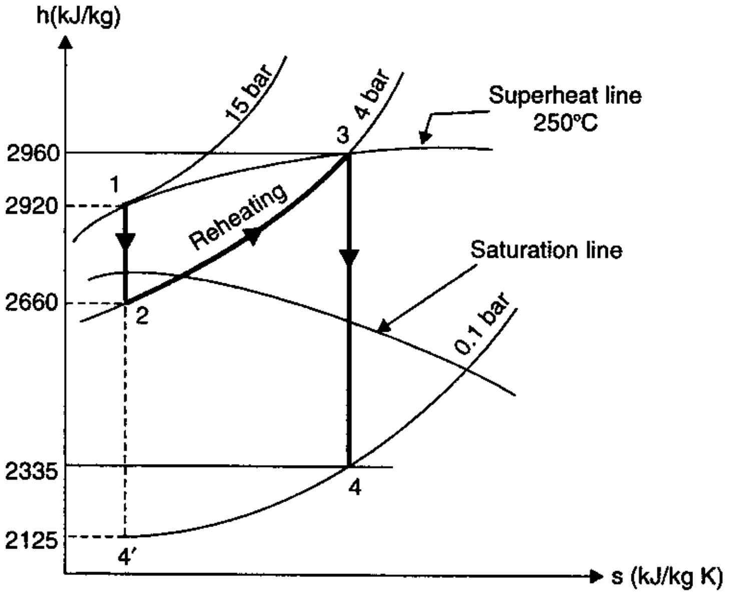


Fig. 2.17

Work done per kg of steam,

$$W = \text{Total heat drop}$$

$$= [(h_1 - h_2) + (h_3 - h_4)] \text{ kJ/kg} \quad ...(i)$$

Amount of heat supplied during process of reheat,

$$h_{\text{reheat}} = (h_3 - h_2)\ \text{kJ/kg} \qquad \ldots(ii)$$

From Mollier diagram or h-s chart,

$$h_1 = 2920\ \text{kJ/kg},\ h_2 = 2660\ \text{kJ/kg}$$
$$h_3 = 2960\ \text{kJ/kg},\ h_4 = 2335\ \text{kJ/kg}$$

Now, by putting the values in eqns. (*i*) and (*ii*), we get

$$W = (2920 - 2660) + (2960 - 2335)$$
$$= \mathbf{885\ kJ/kg.}$$

Hence *work done per kg of steam* = **885 kJ/kg. (Ans.)**

Amount of heat supplied during reheat,

$$h_{\text{reheat}} = (2960 - 2660) = \mathbf{300\ kJ/kg.} \quad \textbf{(Ans.)}$$

If the expansion would have been continuous without reheating *i.e.*, 1 to 4′, the work output is given by

$$W_1 = h_1 - h_4'$$

From Mollier diagram, $h_4' = 2125$ kJ/kg

$\therefore$ $W_1 = 2920 - 2125 =$ **795 kJ/kg. (Ans.)**

Example 2.9. *A turbine is supplied with steam at a pressure of 32 bar and a temperature of 410°C. The steam then expands isentropically to a pressure of 0.08 bar. Find the dryness fraction at the end of expansion and thermal efficiency of the cycle.*

If the steam is reheated at 5.5 bar to a temperature of 400°C and then expanded isentropically to a pressure of 0.08 bar, what will be the dryness fraction and thermal efficiency of the cycle ?

Solution. First case. Refer Fig. 2.18.

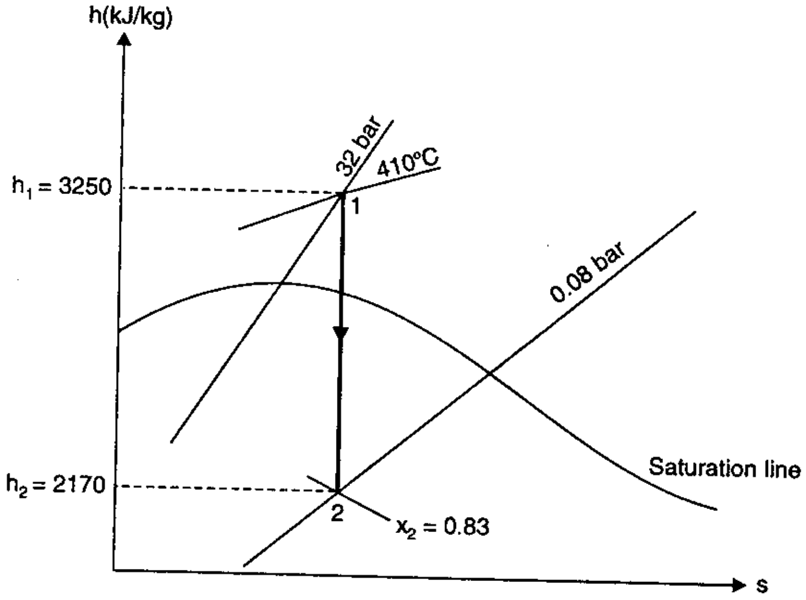


Fig. 2.18

From Mollier chart : $h_1 = 3250$ kJ/kg

$h_2 = 2170$ kJ/kg

Heat drop (or work done) $= h_1 - h_2$

$= 3250 - 2170 = 1080$ kJ/kg

Heat supplied $= h_1 - h_{f_2}$

$= 3250 - 173.9$ $[h_{f_2} = 173.9$ kJ/kg at 0.08 bar]

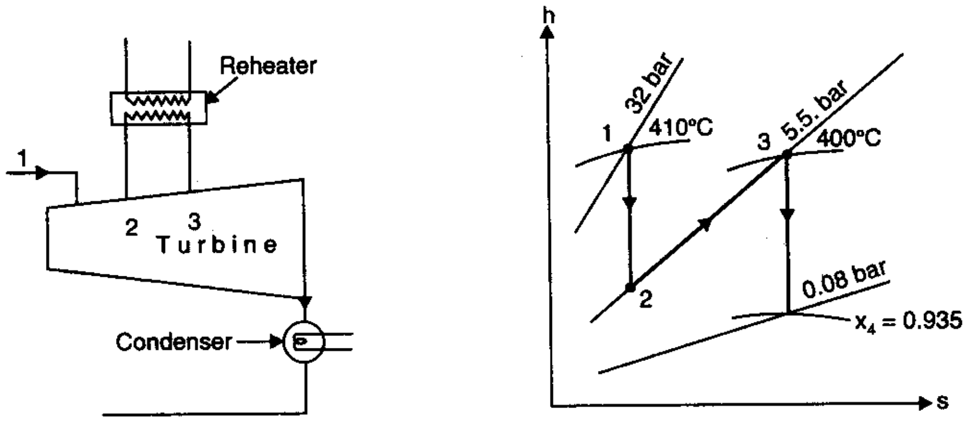
$= 3076.1$ kJ/kg

Thermal efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{1080}{3076.1} = \mathbf{0.351}$ **or 35.1%. (Ans.)**

Exhaust steam condition, x_2

= 0.83 (From Mollier chart). (Ans.)

Second case. Refer Fig. 2.19 (*b*).



(a) (b)

Fig. 2.19

From Mollier chart :

$h_1 = 3250$ kJ/kg
$h_2 = 2807$ kJ/kg
$h_3 = 3263$ kJ/kg
$h_4 = 2426$ kJ/kg.

Work done $= (h_1 - h_2) + (h_3 - h_4) = (3250 - 2807) + (3263 - 2426) = 12080$ kJ/kg

Heat supplied $= (h_1 - h_{f_4}) + (h_3 - h_2)$

$= (3250 - 173.9) + (3263 - 2807) = 3532$ kJ/kg

Thermal efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{1280}{3532} = \mathbf{0.362}$ **or 36.2%. (Ans.)**

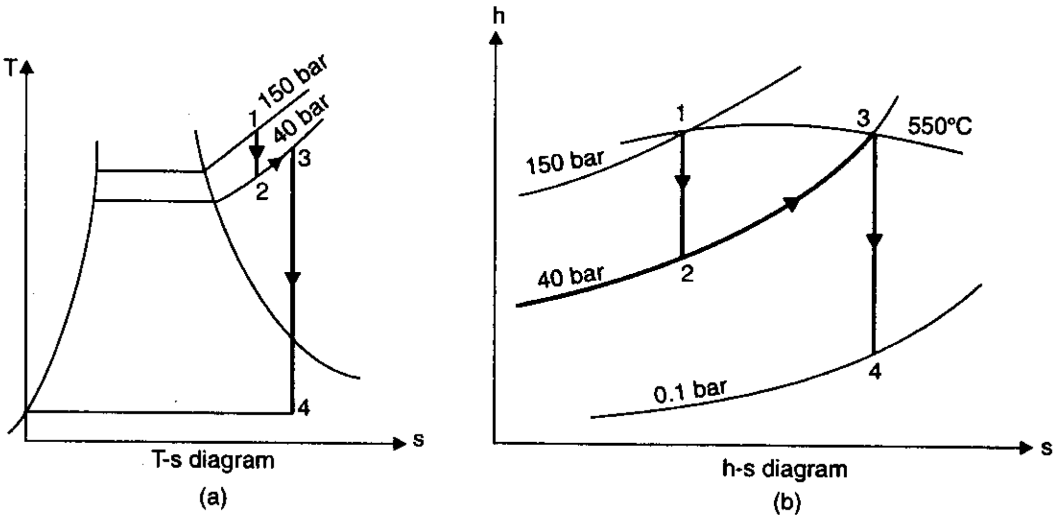
Condition of steam at the exhaust,

$\mathbf{x_4 = 0.935}$ **[From Mollier chart]. (Ans.)**

Example 2.10. *A steam power plant operates on a theoretical reheat cycle. Steam at boiler at 150 bar, 550°C expands through the high pressure turbine. It is reheated at a constant pressure of 40 bar to 550°C and expands through the low pressure turbine to a condenser at 0.1 bar. Draw T-s and h-s diagrams. Find :*

(i) Quality of steam at turbine exhaust ; *(ii) Cycle efficiency ;*
(iii) Steam rate in kg/kWh.

Solution. Refer Fig. 2.20.



T-s diagram
(a)

h-s diagram
(b)

Fig. 2.20

From Mollier diagram (*h-s* diagram) :

$h_1 = 3450$ kJ/kg ; $h_2 = 3050$ kJ/kg ; $h_3 = 3560$ kJ/kg ; $h_4 = 2300$ kJ/kg

h_{f_4} (from steam tables, at 0.1 bar) = 191.8 kJ/kg

(*i*) **Quality of steam at turbine exhaust, x_4 :**

$x_4 = 0.88$ (From Mollier diagram)

(*ii*) **Cycle efficiency, η_{cycle} :**

$$\eta_{\text{cycle}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)}$$

$$= \frac{(3450 - 3050) + (3560 - 2300)}{(3450 - 191.8) + (3560 - 3050)} = \frac{1660}{3768.2} = \mathbf{0.4405 \text{ or } 44.05\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Steam rate in kg/kWh :**

$$\text{Steam rate} = \frac{3600}{(h_1 - h_2) + (h_3 - h_4)} = \frac{3600}{(3450 - 3050) + (3560 - 2300)}$$

$$= \frac{3600}{1660} = \mathbf{2.17 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

2.7. REGENERATIVE CYCLE

In the Rankine cycle it is observed that the condensate which is fairly at low temperature has an irreversible mixing with hot boiler water and this results in decrease of cycle efficiency. Methods are, therefore, adopted to heat the feed water from the hot well of condenser irreversibly by interchange of heat within the system and thus improving the cycle efficiency. This heating method is called regenerative feed heat and the cycle is called *regenerative cycle*.

The principle of regeneration can be practically utilised by extracting steam from the turbine at several locations and supplying it to the regenerative heaters. The resulting cycle is known as *regenerative or bleeding cycle.* The heating arrangement comprises of : (*i*) For medium capacity turbines—not more than 3 heaters ; (*ii*) For high pressure high capacity turbines—not more than 5 to 7 heaters ; and (*iii*) For turbines of supercritical parameters 8 to 9 heaters. *The most advantageous condensate heating temperature is selected depending on the turbine throttle conditions and this determines the number of heaters to be used.* The final condensate heating temperature is kept 50 to 60°C below the boiler saturated steam temperature so as to prevent evaporation of water in the feed mains following a drop in the boiler drum pressure. The conditions of steam bled for each heater are selected so that the temperature of saturated steam will be 4 to 10°C higher than the final condensate temperature.

Fig. 2.21 (*a*) shows a diagrammatic layout of a condensing steam power plant in which a surface condenser is used to condense all the steam that is not extracted for feed water heating. The turbine is double extracting and the boiler is equipped with a superheater. The cycle diagram (*T-s*) would appear as shown in Fig. 2.21 (*b*). This arrangement constitutes a *regenerative cycle.*

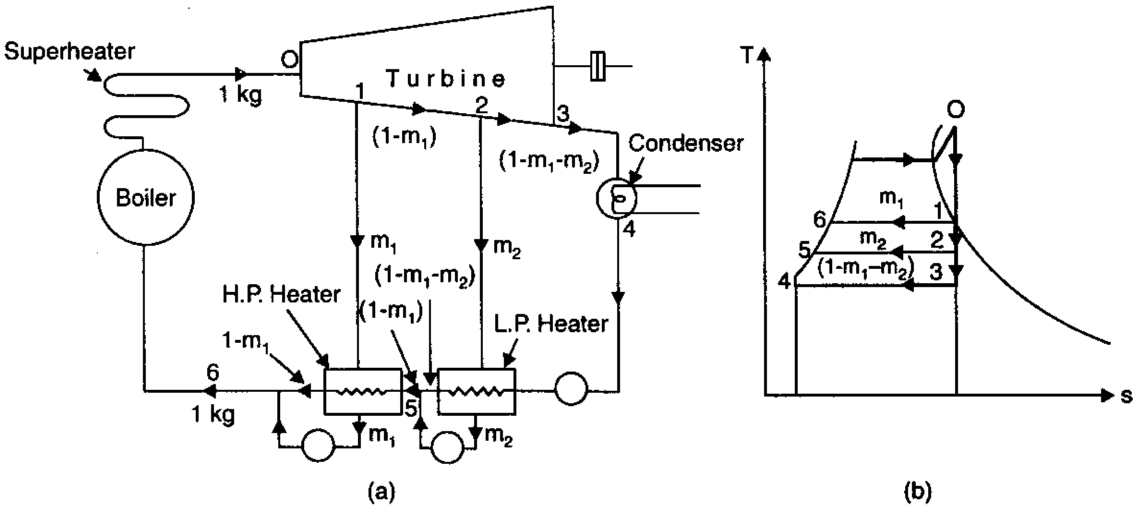


Fig. 2.21. Regenerative cycle.

Let, m_1 = kg of high pressure (H.P.) steam per kg of steam flow,

m_2 = kg of low pressure (L.P.) steam extracted per kg of steam flow

$(1 - m_1 - m_2)$ = kg of steam entering condenser per kg of steam flow.

Energy/Heat balance equation for H.P. heater :

$$m_1 (h_1 - h_{f_6}) = (1 - m_1)(h_{f_6} - h_{f_5})$$

or $$m_1[(h_1 - h_{f_6}) + (h_{f_6} - h_{f_5})] = (h_{f_6} - h_{f_5}) \quad \text{or} \quad m_1 = \frac{h_{f_6} - h_{f_5}}{h_1 - h_{f_5}} \qquad ...(2.11)$$

Energy/Heat balance equation for L.P. heater :

$$m_2 (h_2 - h_{f_5}) = (1 - m_1 - m_2)(h_{f_5} - h_{f_3})$$

or $$m_2[(h_2 - h_{f_5}) + (h_{f_5} - h_{f_3})] = (1 - m_1)(h_{f_5} - h_{f_3})$$

or $$m_2 = \frac{(1 - m_1)(h_{f_5} - h_{f_3})}{(h_2 - h_{f_3})} \qquad ...(2.12)$$

All enthalpies may be determined ; therefore m_1 and m_2 may be found. The maximum temperature to which the water can be heated is dictated by that of bled steam. The condensate from the bled steam is added to feed water.

Neglecting pump work :

The heat supplied externally in the cycle

$$= (h_0 - h_{f_6})$$

Isentropic work done $= m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2) (h_0 - h_3)$

The thermal efficiency of regenerative cycle is

$$\eta_{\text{thermal}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2) (h_0 - h_3)}{(h_0 - h_{f_6})} \qquad \text{...(2.13)}$$

[The work done by the turbine may also be calculated by summing up the products of the steam flow and the corresponding heat drop in the turbine stages.

i.e., Work done $= (h_0 - h_1) + (1 - m_1) (h_1 - h_2) + (1 - m_1 - m_2) (h_2 - h_3)$]

Advantages of Regenerative cycle over Simple Rankine cycle :

1. The heating process in the boiler tends to become reversible.

2. The thermal stresses set up in the boiler are minimised. This is due to the fact that temperature ranges in the boiler are reduced.

3. The thermal efficiency is improved because the average temperature of heat addition to the cycle is increased.

4. Heat rate is reduced.

5. The blade height is less due to the reduced amount of steam passed through the low pressure stages.

6. Due to many extractions there is an improvement in the turbine drainage and it reduces erosion due to moisture.

7. A small size condenser is required.

Disadvantages :

1. The plant becomes more complicated.

2. Because of addition of heaters greater maintenance is required.

3. For given power a large capacity boiler is required.

4. The heaters are costly and the gain in thermal efficiency is not much in comparison to the heavier costs.

Note. In the absence of precise information (regarding actual temperature of the feed water entering and leaving the heaters and of the condensate temperatures) the following assumptions should always be made while doing calculations :

1. Each heater is ideal and bled steam just condenses.

2. The feed water is heated to saturation temperature at the pressure of bled steam.

3. Unless otherwise stated the work done by the pumps in the system is considered negligible.

4. There is equal temperature rise in all the heaters (usually 10°C to 15°C).

☞**Example 2.11.** *In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Find :*

(*i*) *The efficiency and the steam rate of the cycle.*

(*ii*) *The increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration).*

Pump work may be neglected.

Solution. Fig. 2.22 shows the flow, *T-s* and *h-s* diagrams.

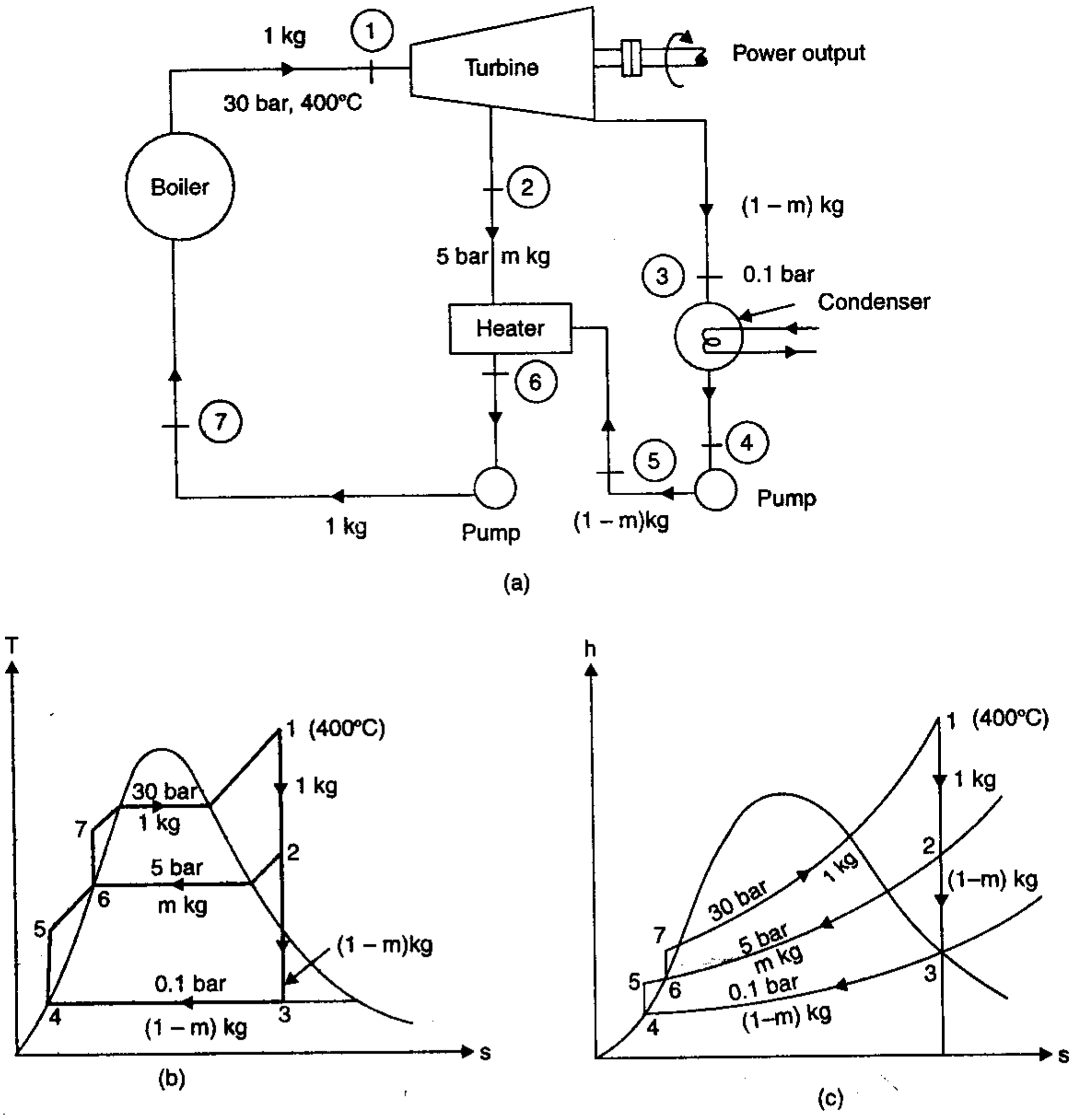


Fig. 2.22

From steam tables :

At 30 bar, 400°C : $h_1 = 3230.9$ kJ/kg, $s_1 = 6.921$ kJ/kg K $= s_2 = s_3$,

At 5 bar : $s_f = 1.8604$ kJ/kg K, $s_g = 6.8192$ kJ/kg K, $h_f = 640.1$ kJ/kg

Since $s_2 > s_g$, the state 2 must lie in the superheated region. From the table for superheated steam $t_2 = 172$°C, $h_2 = 2796$ kJ/kg.

At 0.1 bar : $s_f = 0.649$ kJ/kg K, $s_{f_g} = 7.501$ kJ/kg K,

$h_f = 191.8$ kJ/kg K, $h_{f_g} = 2392.8$ kJ/kg K.

Now, $s_2 = s_3$

i.e., $6.921 = s_{f_3} + x_3\, s_{fg_3} = 0.649 + x_3 \times 7.501$

$$\therefore \quad x_3 = \frac{6.921 - 0.649}{7.501} = 0.836$$

$$\therefore \quad h_3 = h_{f_3} + x_3\, h_{fg_3} = 191.8 + 0.836 \times 2392.8 = 2192.2 \text{ kJ/kg}$$

Since pump work is neglected

$$h_{f_4} = 191.8 \text{ kJ/kg} = h_{f_5}$$

$$h_{f_6} = 640.1 \text{ (at 5 bar)} = h_{f_7}$$

Energy balance for heater gives

$$m\,(h_2 - h_{f_6}) = (1 - m)\,(h_{f_6} - h_{f_5})$$

$$m\,(2796 - 640.1) = (1 - m)\,(640.1 - 191.8) = 448.3\,(1 - m)$$

$$2155.9\, m = 448.3 - 448.3\, m$$

$$\therefore \quad m = 0.172 \text{ kg}$$

$\therefore$ Turbine work, $W_T = (h_1 - h_2) + (1 - m)\,(h_2 - h_3)$

$= (3230.9 - 2796) + (1 - 0.172)\,(2796 - 2192.2)$

$= 434.9 + 499.9 = 934.8$ kJ/kg

Heat supplied, $Q_1 = h_1 - h_{f_6} = 3230.9 - 640.1 = 2590.8$ kJ/kg.

(*i*) $$\eta_{\text{cycle}} = \frac{W_T}{Q_1} = \frac{934.8}{2590.8} = \mathbf{0.3608 \text{ or } 36.08\%.} \quad \textbf{(Ans.)}$$

$$\textit{Steam rate} = \frac{3600}{934.8} = \mathbf{3.85 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

(*ii*) $$T_{m_1} = \frac{h_1 - h_{f_7}}{s_1 - s_7} = \frac{2590.8}{6.921 - 1.8604} = 511.9 \text{ K} = 238.9\text{°C}.$$

T_{m_1} (without regeneration)

$$= \frac{h_1 - h_{f_4}}{s_1 - s_4} = \frac{3230.9 - 191.8}{6.921 - 0.649} = \frac{3039.1}{6.272} = 484.5 \text{ K} = 211.5\text{°C}.$$

Increase in T_{m_1} due to regeneration

$= 238.9 - 211.5 = $ **27.4°C. (Ans.)**

W_T (without regeneration)

$= h_1 - h_3 = 3230.9 - 2192.2 = 1038.7$ kJ/kg

Steam rate without regeneration

$$= \frac{3600}{1038.7} = 3.46 \text{ kg/kWh}$$

∴ *Increase in steam rate due to regeneration*

$$= 3.85 - 3.46 = \mathbf{0.39\ kg/kWh.} \quad \textbf{(Ans.)}$$

$$\eta_{\text{cycle}} \text{ (without regeneration)} = \frac{h_1 - h_3}{h_1 - h_{f_4}} = \frac{1038.7}{3230.9 - 191.8} = \mathbf{0.3418 \text{ or } 34.18\%.}$$

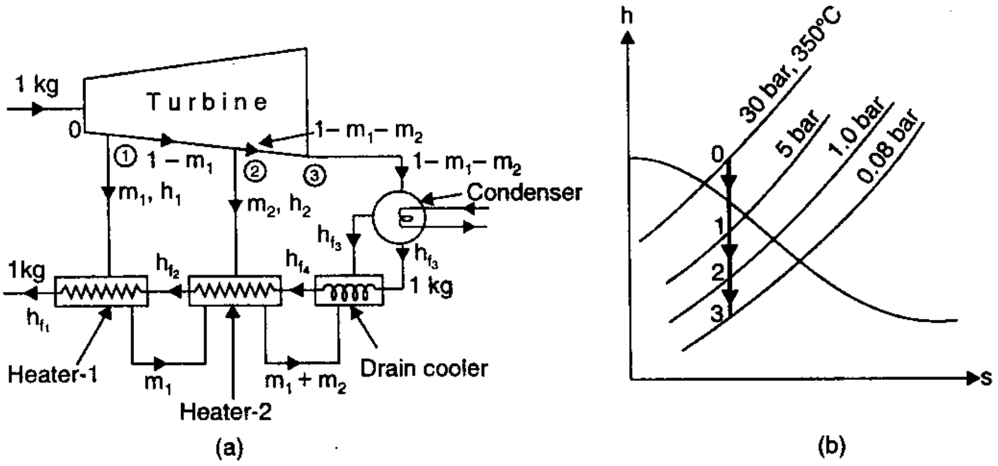
Increase in cycle efficiency due to regeneration

$$= 36.08 - 34.18 = \mathbf{1.9\%.} \quad \textbf{(Ans.)}$$

Example 2.12. *Steam is supplied to a turbine at 30 bar and 350°C. The turbine exhaust pressure is 0.08 bar. The main condensate is heated regeneratively in two stages by steam bled from the turbine at 5 bar and 1.0 bar respectively. Calculate masses of steam bled off at each pressure per kg of steam entering the turbine and the theoretical thermal efficiency of the cycle.*

Make any suitable assumptions. Assume drain cooler is also used.

Solution. Refer Fig. 2.23.



(a) (b)

Fig. 2.23

The following ***assumptions*** are made :

1. The condensate is heated to the saturation temperature in each heater.

2. The drain water from H.P. (high pressure) heater passes into the steam space of the L.P. (low pressure) heater without loss of heat.

3. The combined drains from the L.P. heater are cooled in a drain cooler to the condenser temperature.

4. The expansion of the steam in the turbine is adiabatic and frictionless.

Enthalpy at 30 bar, 350°C, $h_0 = 3115.3$ kJ/kg.

After adiabatic expansion (from Mollier chart)

Enthalpy at 5 bar, $h_1 = 2720$ kJ/kg

Enthalpy at 1.0 bar, $h_2 = 2450$ kJ/kg

Enthalpy at 0.08 bar, $h_3 = 2120$ kJ/kg

From steam tables : $h_{f_1} = 640.1$ kJ/kg (at 5.0 bar)

$h_{f_2} = 417.5$ kJ/kg (at 1.0 bar)

$h_{f_3} = 173.9$ kJ/kg (at 0.08 bar)

At heater No. 1 :

$$m_1 h_1 + h_{f_2} = m_1 h_{f_1} + h_{f_1}$$

$$\mathbf{m_1} = \frac{h_{f_1} - h_{f_2}}{h_1 - h_{f_1}} = \frac{640.1 - 417.5}{2720 - 640.1} = \textbf{0.107 kJ/kg of entering steam.}$$

(Ans.)

At heater No. 2 :

$$m_2 h_2 + m_1 h_{f_1} + h_{f_4} = (m_1 + m_2)\, h_{f_2} + h_{f_2} \quad \text{...}(i)$$

At drain cooler :

$$(m_1 + m_2)\, h_{f_2} + h_{f_3} = h_{f_4} + (m_1 + m_2)\, h_{f_3}$$

$$\therefore \quad h_{f_4} = (m_1 + m_2)(h_{f_2} - h_{f_3}) + h_{f_3} \quad \text{...}(ii)$$

Inserting the value of h_{f_4} in eqn. (*i*), we get

$$m_2 h_2 + m_1 h_{f_1} + (m_1 + m_2)(h_{f_2} - h_{f_3}) + h_{f_3} = (m_1 + m_2)\, h_{f_2} + h_{f_2}$$

$$m_2 h_2 + m_1 h_{f_1} + (m_1 + m_2) h_{f_2} - (m_1 + m_2) h_{f_3} + h_{f_3} = (m_1 + m_2) h_{f_2} + h_{f_2}$$

$$m_2 h_2 + m_1 h_{f_1} - m_1 h_{f_3} - m_2 h_{f_3} + h_{f_3} = h_{f_2}$$

$$m_2 (h_2 - h_{f_3}) = (h_{f_2} - h_{f_3}) - m_1 (h_{f_1} - h_{f_3})$$

$$\therefore \quad \mathbf{m_2} = \frac{(h_{f_2} - h_{f_3}) - m_1 (h_{f_1} - h_{f_3})}{(h_2 - h_{f_3})}$$

$$= \frac{(417.5 - 173.9) - 0.107\,(640.1 - 173.9)}{(2450 - 173.9)}$$

$$= \frac{193.7}{2276.1} = \textbf{0.085 kJ/kg.} \quad \textbf{(Ans.)}$$

Work done

$$= 1\,(h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)$$

$$= 1\,(3115.3 - 2720) + (1 - 0.107)(2720 - 2450) + (1 - 0.107 - 0.085)(2450 - 2120)$$

$$= 395.3 + 241.11 + 266.64 = 903.05 \text{ kJ/kg}$$

Heat supplied/kg

$$= h_0 - h_{f_1}$$

$$= 3115.3 - 640.1 = 2475.2 \text{ kJ/kg}$$

∴ *Thermal efficiency of the cycle*

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{903.05}{2475.2} = \textbf{0.3648 or 36.48\%.} \quad \textbf{(Ans.)}$$

☞**Example 2.13.** *Steam at a pressure of 20 bar and 250°C enters a turbine and leaves it finally at a pressure of 0.05 bar. Steam is bled off at pressures of 5.0, 1.5 and 0.3 bar. Assuming (i) that the condensate is heated in each heater upto the saturation temperature of the steam in that heater, (ii) that the drain water from each heater is cascaded through a trap into the next heater on the* *low pressure side of it, (iii) that the combined drains from the heater operating at 0.3 bar are cooled in a drain cooler to condenser temperature, calculate the following :*

(*i*) *Mass of bled steam for each heater per kg of steam entering the turbine.*

(*ii*) *Thermal efficiency of the cycle.*

(*iii*) *Thermal efficiency of the Rankine cycle.*

(*iv*) *Theoretical gain due to regenerative feed heating.*

(*v*) *Steam consumption in kg/kWh with or without regenerative feed heating.*

(*vi*) *Quantity of steam passing through the last stage nozzle of a 50000 kW turbine with and without regenerative feed heating.*

Solution. Refer Fig. 2.24 (*a*), (*b*).

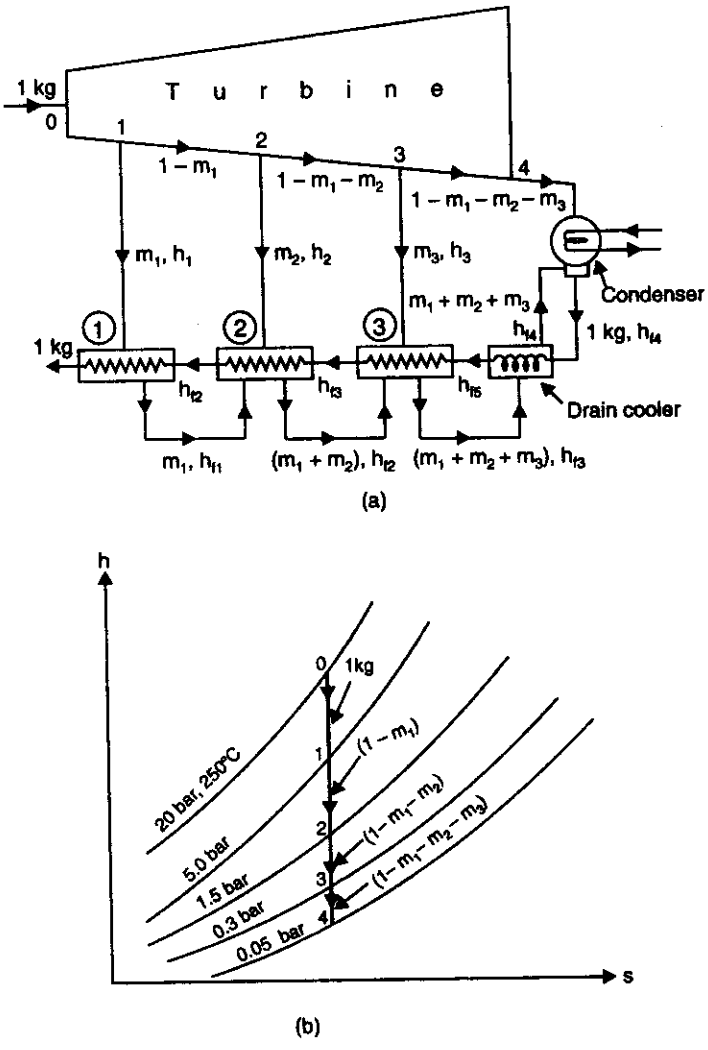


Fig. 2.24

$h_0 = 2905$ kJ/kg, $h_1 = 2600$ kJ/kg, $h_2 = 2430$ kJ/kg

$h_3 = 2210$ kJ/kg, $h_4 = 2000$ kJ/kg

From steam tables :

At 5 bar : $h_{f_1} = 640.1$ kJ/kg

At 1.5 bar : $h_{f_2} = 467.1$ kJ/kg

At 0.3 bar : $h_{f_3} = 289.3$ kJ/kg

At 0.05 bar : $h_{f_4} = 137.8$ kJ/kg.

(*i*) **Mass of bled steam for each heater per kg of steam :**

Using heat balance equation :

At heater No. 1 :

$$m_1 h_1 + h_{f_2} = m_1 h_{f_1} + h_{f_1}$$

$$\therefore \quad \mathbf{m_1} = \frac{h_{f_1} - h_{f_2}}{h_1 - h_{f_1}} = \frac{640.1 - 467.1}{2600 - 640.1}$$

= **0.088 kJ/kg of entering steam. (Ans.)**

At heater No. 2 :

$$m_2 h_2 + h_{f_3} + m_1 h_{f_1} = h_{f_2} + (m_1 + m_2) h_{f_2}$$

$$\mathbf{m_2} = \frac{(h_{f_2} + h_{f_3}) - m_1 (h_{f_1} - h_{f_2})}{(h_2 - h_{f_2})}$$

$$= \frac{(467.1 - 289.3) - 0.088\,(640.1 - 467.1)}{(2430 - 467.1)} = \frac{162.57}{1962.9}$$

= **0.0828 kJ/kg of entering steam. (Ans.)**

At heater No. 3 :

$$m_3 h_3 + h_{f_5} + (m_1 + m_2) h_{f_2} = h_{f_3} + (m_1 + m_2 + m_3) h_{f_3} \qquad ...(i)$$

At drain cooler :

$$(m_1 + m_2 + m_3) h_{f_3} + h_{f_4} = h_{f_5} + (m_1 + m_2 + m_3) h_{f_4}$$

$$\therefore \quad h_{f_5} = (m_1 + m_2 + m_3)\,(h_{f_3} - h_{f_4}) + h_{f_4} \qquad ...(ii)$$

Inserting the value of h_{f_5} in eqn. (*i*), we get

$$m_3 h_3 + (m_1 + m_2 + m_3)\,(h_{f_3} - h_{f_4}) + h_{f_4} + (m_1 + m_2)\, h_{f_2} = h_{f_3} + (m_1 + m_2 + m_3) h_{f_3}$$

$$\therefore \quad \mathbf{m_3} = \frac{(h_{f_3} - h_{f_4}) - (m_1 + m_2)(h_{f_2} - h_{f_4})}{h_3 - h_{f_4}}$$

$$= \frac{(289.3 - 137.8) - (0.088 + 0.0828)(467.1 - 137.8)}{(2210 - 137.8)}$$

$= \dfrac{151.5 - 56.24}{2072.2}$ = **0.046 kJ/kg of entering steam. (Ans.)**

Work done/kg (neglecting pump work)

$$= (h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3) + (1 - m_1 - m_2 - m_3)(h_3 - h_4)$$

$$= (2905 - 2600) + (1 - 0.088)(2600 - 2430) + (1 - 0.088 - 0.0828)(2430 - 2210)$$
$$+ (1 - 0.088 - 0.0828 - 0.046)(2210 - 2000)$$

$= 305 + 155.04 + 182.42 + 164.47 = 806.93$ kJ/kg

Heat supplied/kg $= h_0 - h_{f_1} = 2905 - 640.1 = 2264.9$ kJ/kg.

(*ii*) **Thermal efficiency of the cycle :**

$$\eta_{\text{Thermal}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{806.93}{2264.9} = \mathbf{0.3563 \text{ or } 35.63\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Thermal efficiency of Rankine cycle :**

$$\eta_{\text{Rankine}} = \frac{h_0 - h_4}{h_0 - h_{f_4}} = \frac{2905 - 2000}{2905 - 137.8} = \mathbf{0.327 \text{ or } 32.7\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Theoretical gain due to regenerative feed heating**

$$= \frac{35.63 - 32.7}{35.63} = \mathbf{0.0822 \text{ or } 8.22\%.} \quad \textbf{(Ans.)}$$

(*v*) **Steam consumption with regenerative feed heating**

$$= \frac{1 \times 3600}{\text{Work done / kg}} = \frac{1 \times 3600}{806.93} = \mathbf{4.46 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

Steam consumption without regenerative feed heating

$$= \frac{1 \times 3600}{\text{Work done / kg without regeneration}} = \frac{1 \times 3600}{h_0 - h_4}$$

$$= \frac{1 \times 3600}{2905 - 2000} = \mathbf{3.97 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

(*vi*) **Quantity of steam passing through the last stage of a 50000 kW turbine with regenerative feed-heating**

$= 4.46 (1 - m_1 - m_2 - m_3) \times 50000$

$= 4.46 (1 - 0.088 - 0.0828 - 0.046) \times 50000 =$ **174653.6 kg/h. (Ans.)**

Same without regenerative arrangement

$= 3.97 \times 50000 =$ **198500 kg/h. (Ans.)**

2.8. BINARY VAPOUR CYCLE

Carnot cycle gives the highest thermal efficiency which is given by $\frac{T_1 - T_2}{T_1}$. To approach this cycle in an actual engine it is necessary that whole of heat must be supplied at constant temperature T_1 and rejected at T_2. This can be achieved only by using a vapour in the wet field but not in the superheated. The efficiency depends on temperature T_1 since T_2 is fixed by the natural sink to which heat is rejected. This means that T_1 should be as large as possible, consistent with the vapour being saturated.

If we use steam as the working medium the temperature rise is accompanied by rise in pressure and at critical temperature of 374.15°C the pressure is as high as 226 bar (approx.) which will create many difficulties in design, operation and control. It would be desirable to use some fluid other than steam which has more desirable thermodynamic properties than water. An ideal fluid for this purpose should have a very high critical temperature combined with low pressure. Mercury, diphenyl-oxide and similar compounds, aluminium bromide and zinc ammonium chloride are fluids which possess the required properties in varying degrees. Mercury is the only working fluid which has been successfully used in practice. It has high critical temperature (588.4°C) and correspondingly low critical pressure (21 bar abs.). *The mercury alone cannot be used as its saturation temperature at* *atmospheric pressure is high (357°C). Hence binary vapour cycle is generally used to increase the overall efficiency of the plant.* Two fluids (mercury and water) are used in cascade in the binary cycle for production of power.

The *few more properties* required for an ideal binary fluid used in high temperature limit are listed below :

1. It should have high critical temperature at reasonably low pressure.
2. It should have high heat of vaporisation to keep the weight of fluid in the cycle to minimum.
3. Freezing temperature should be below room temperature.
4. It should have chemical stability throughout the working cycle.
5. It must be non-corrosive to the metals normally used in power plants.
6. It must have an ability to wet the metal surfaces to promote the heat transfer.
7. The vapour pressure at a desirable condensation temperature should be nearly atmospheric which will eliminate requirement of power for maintenance of vacuum in the condenser.
8. After expansion through the primemover the vapour should be nearly saturated so that a desirable heat transfer co-efficient can be obtained which will reduce the size of the condenser required.
9. It must be available in large quantities at reasonable cost.
10. It should not be toxic and, therefore, dangerous to human life.

Although mercury does not have all the required properties, it is more favourable than any other fluid investigated. It is most stable under all operating conditions.

Although, mercury does not cause any corrosion to metals, but it is extremely dangerous to human life, therefore, elaborate precautions must be taken to prevent the escape of vapour. The major *disadvantage* associated with mercury is that it *does not wet surface of the metal and forms a serious resistance to heat flow.* This difficulty can be considerably reduced by adding magnesium and titanium (2 parts in 100000 parts) in mercury.

Thermal properties of mercury :

Mercury fufills practically all the *desirable thermodynamic properties* stated above.

1. Its freezing point is – 3.3°C and boiling point is – 354.4°C at atmospheric pressure.
2. The pressure required when the temperature of vapour is 540°C is only 12.5 bar (approx.) and, therefore, heavy construction is not required to get high initial temperature.
3. Its liquid saturation curve is very steep, approaching the isentropic of the Carnot cycle.
4. It has no corrosive or erosive effects upon metals commonly used in practice.
5. Its critical temperature is so far removed from any possible upper temperature limit with existing metals as to cause no trouble.

Some *undesirable properties* of mercury are listed below :

1. Since the latent heat of mercury is quite low over a wide range of desirable condensation temperatures, therefore, several kg of mercury must be circulated per kg of water evaporated in binary cycle.
2. The cost is a considerable item as the quantity required is 8 to 10 times the quantity of water circulated in binary system.
3. Mercury vapour in large quantities is poisonous, therefore, the system must be perfect and tight.

Fig. 2.25 shows the schematic line diagram of binary vapour cycle using mercury and water as working fluids. The processes are represented on *T-s* diagram as shown in Fig. 2.26.

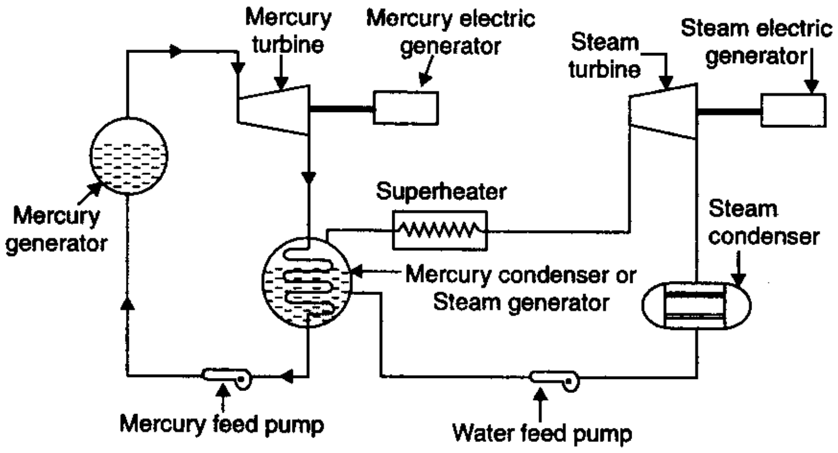


Fig. 2.25. Line diagram of binary vapour cycle.

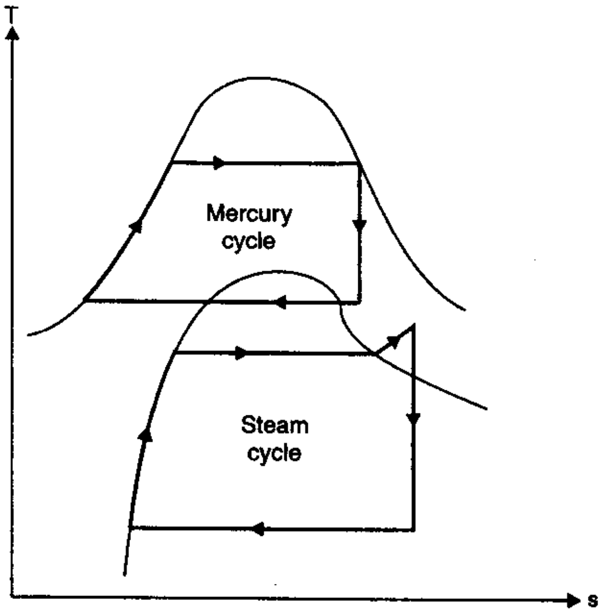


Fig. 2.26. Binary vapour cycle on *T-s* diagram.

Analysis of Binary vapour cycle :

h_{hg_1} = Heat supplied per kg of Hg (mercury) vapour formed in the mercury boiler.

h_{hg_2} = Heat lost by one kg of Hg vapour in the mercury condenser.

h_s = Heat given per kg of steam generated in the mercury condenser or steam boiler.

W_{hg} = Work done per kg of Hg in the cycle.

W_s = Work done per kg of steam in the steam cycle.

η_s = Thermal efficiency of the steam cycle.

η_{hg} = Thermal efficiency of the Hg cycle.

m = Mass of Hg in the Hg cycle per kg of steam circulated in the steam cycle.

The heat losses to the surroundings, in the following analysis, are neglected and steam generated is considered one kg and Hg in the circuit is m kg per kg of water in the steam cycle.

Heat supplied in the Hg boiler,

$$h_t = m \times h_{hg_1} \quad \text{...(2.14)}$$

Work done in the mercury cycle

$$= m \cdot W_{hg} \quad \text{...(2.15)}$$

Work done in the steam cycle

$$= 1 \times W_s \quad \text{...(2.16)}$$

Total work done in the binary cycle is given by

$$W_t = m\, W_{hg} + W_s \quad \text{...(2.17)}$$

∴ Overall efficiency of the binary cycle is given by

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_t}{h_t} = \frac{mW_{hg} + W_s}{m\, h_{hg_1}} \quad \text{...(2.18)}$$

Thermal efficiency of the mercury cycle is given by

$$\eta_{hg} = \frac{mW_{hg}}{m\, h_{hg_1}}$$

$$= \frac{W_{hg}}{h_{hg_1}} = \frac{h_{hg_1} - h_{hg_2}}{h_{hg_1}} = 1 - \frac{h_{hg_2}}{h_{hg_1}} \quad \text{...(2.19)}$$

$$= \frac{m\, h_{hg_1} - h_s}{m\, h_{hg_1}} = 1 - \frac{1}{m} \cdot \frac{h_s}{h_{hg_1}} \quad \text{...(2.20)}$$

Heat lost by mercury vapour = Heat gained by steam

$$\therefore \quad m\, h_{hg_2} = 1 \times h_s \quad \text{...(2.21)}$$

Substituting the value of m from equation (2.21) into equation (2.20), we get

$$\eta_{hg} = 1 - \frac{h_{hg_2}}{h_{hg_1}} \quad \text{...(2.22)}$$

The thermal efficiency of the steam cycle is given by

$$\eta_s = \frac{W_s}{h_s} = \frac{h_{s_1} - h_{s_2}}{h_{s_1}} = \frac{h_{s_1} - h_{s_2}}{m\, h_{hg_2}} \quad \text{...(2.23)}$$

From the equations. (2.18), (2.20), (2.21), (2.22) and (2.23), we get

$$\eta = \eta_{hg}\,(1 - \eta_s) + \eta_s \quad \text{...(2.24)}$$

To solve the problems eqns. (2.19), (2.23), (2.24) are used.

In the design of binary cycle, another important problem is the limit of exhaust pressure of the mercury (location of optimum exhaust pressure) which will provide maximum work per kg of Hg circulated in the system and high thermal efficiency of the cycle. It is not easy to decide as number of controlling factors are many.

☞**Example 2.14.** *A binary vapour cycle operates on mercury and steam. Standard mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhausts at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.*

(*i*) *Determine the overall efficiency of the cycle.*

(*ii*) *If 48000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?*

(*iii*) *Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?*

(*iv*) *If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler and if the internal efficiencies of the mercury and steam turbines are 0.84 and 0.88 respectively, calculate the overall efficiency of the cycle.*

The properties of standard mercury are given below :

p (*bar*)	*t* (°*C*)	h_f (*kJ/kg*)	h_g (*kJ/kg*)	s_f (*kJ/kg K*)	s_g (*kJ/kg K*)	v_f (m^3/kg)	v_g (m^3/kg)
4.5	*450*	*62.93*	*355.98*	*0.1352*	*0.5397*	79.9×10^{-6}	*0.068*
0.04	*216.9*	*29.98*	*329.85*	*0.0808*	*0.6925*	76.5×10^{-6}	*5.178.*

Solution. The binary vapour cycle is shown in Fig. 2.27.

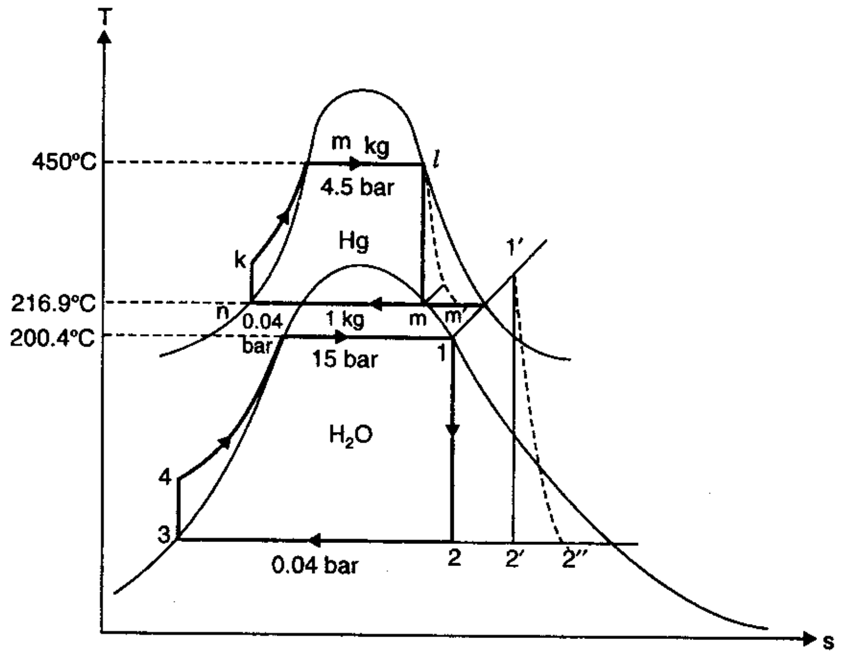


Fig. 2.27

Mercury cycle :

$$h_l = 355.98 \text{ kJ/kg}$$

$$s_l = 0.5397 = s_m = s_f + x_m \, s_{fg}$$

or

$$0.5397 = 0.0808 + x_m = (0.6925 - 0.0808)$$

$$\therefore \quad x_m = \frac{(0.5397 - 0.0808)}{(0.6925 - 0.0808)} = 0.75$$

$$h_m = h_f + x_m\, h_{fg} = 29.98 + 0.75 \times (329.85 - 29.98)$$
$$= 254.88 \text{ kJ/kg}$$

Work obtained from mercury turbine

$$(W_T)_{Hg} = h_l - h_m = 355.98 - 254.88 = 101.1 \text{ kJ/kg}$$

Pump work in mercury cycle,

$$(W_P)_{Hg} = h_{f_k} - h_{f_n} = 76.5 \times 10^{-6} \times (4.5 - 0.04) \times 100 = 0.0341 \text{ kJ/kg}$$

$$\therefore \quad W_{net} = 101.1 - 0.0341 \simeq 101.1 \text{ kJ/kg}$$

$$Q_1 = h_l - h_{f_k} = 355.98 - 29.98 = 326 \text{ kJ/kg} \qquad (\because\ h_{f_n} \simeq h_{f_k})$$

$$\therefore \quad \eta_{\text{Hg cycle}} = \frac{W_{net}}{Q_1} = \frac{101.1}{326} = 0.31 \text{ or } 31\%.$$

Steam cycle :

At 15 bar : $h_1 = 2789.9$ kJ/kg, $s_1 = 6.4406$ kJ/kg

At 0.04 bar : $h_f = 121.5$ kJ/kg, $h_{f_g} = 2432.9$ kJ/kg,

$s_f = 0.432$ kJ/kg K, $s_{fg_2} = 8.052$ kJ/kg K, $v_f = 0.001\ m^3$/kg

Now, $s_1 = s_2$

$$6.4406 = s_f + x_2\, s_{fg} = 0.423 + x_2 \times 8.052$$

$$\therefore \quad x_2 = \frac{6.4406 - 0.423}{8.052} = 0.747$$

$$h_2 = h_f + x_2\, h_{f_g} = 121.5 + 0.747 \times 2432.9 = 1938.8 \text{ kJ/kg}$$

Work obtained from steam turbine,

$$(W_T)_{steam} = h_1 - h_2 = 2789.9 - 1938.8 = 851.1 \text{ kJ/kg}$$

Pump work in steam cycle,

$$(W_P)_{steam} = h_{f_4} - h_{f_3} = 0.001\,(15 - 0.04) \times 100 = 1.496 \text{ kJ/kg} \simeq 1.5 \text{ kJ/kg}$$

or

$$h_{f_4} = h_{f_3} + 1.5 = 121.5 + 1.5 = 123 \text{ kJ/kg}$$

$$Q_1 = h_1 - h_{f_2} = 2789.9 - 123 = 2666.9 \text{ kJ/kg}$$

$$(W_{net})_{steam} = 851.1 - 1.5 = 849.6 \text{ kJ/kg}$$

$$\therefore \quad \eta_{\text{steam cycle}} = \frac{W_{net}}{Q_1} = \frac{849.6}{2666.6} = 0.318 \text{ or } 31.8\%.$$

(*i*) **Overall efficiency of the binary cycle :**

Overall efficiency of the binary cycle

$$= \eta_{Hg_{cycle}} + \eta_{\text{steam cycle}} - \eta_{Hg_{cycle}} \times \eta_{\text{steam cycle}}$$
$$= 0.31 + 0.318 - 0.31 \times 0.318 = 0.5294 \text{ or } 52.94\%$$

Hence *overall efficiency of the binary cycle* = **52.94%.** **(Ans.)**

$\eta_{overall}$ can also be found out as follows :

Energy balance for a mercury condenser-steam boiler gives :

$$m(h_m - h_{f_n}) = h(h_1 - h_{f_4})$$

where m is the amount of mercury circulating for 1 kg of steam in the bottom cycle

$$\therefore \quad m = \frac{h_1 - h_{f_4}}{h_m - h_{f_n}} = \frac{2666.9}{254.88 - 29.98} = 11.86 \text{ kg}$$

$$(Q_1)_{total} = m\,(h_l - h_{f_k}) = 11.86 \times 326 = 3866.36 \text{ kJ/kg}$$

$$(W_T)_{total} = m\,(h_l - h_m) + (h_1 - h_2)$$

$$= 11.86 \times 101.1 + 851.1 = 2050.1 \text{ kJ/kg}$$

$(W_P)_{total}$ may be *neglected*

$$\eta_{overall} = \frac{W_T}{Q_1} = \frac{2050.1}{3866.36} = 0.53 \text{ or } 53\%.$$

(*ii*) **Flow through mercury turbine :**

If 48000 kg/h of steam flows through the steam turbine, the flow rate of mercury would be

w_{Hg} = 48000 × 11.86 = **569280 kg/h. (Ans.)**

(*iii*) **Useful work in binary vapour cycle :**

Useful work, $(W_T)_{total} = 2050.1 \times 48000 = 9840.5 \times 10^4$ kJ/h

$$= \frac{9840.5 \times 10^4}{3600} = 27334.7 \text{ kW} = \mathbf{27.33\ MW}. \quad \textbf{(Ans.)}$$

(*iv*) **Overall efficiency under new conditions :**

Considering the efficiencies of turbines, we have :

$$(W_T)_{Hg} = h_l - h_m' = 0.84 \times 101.1 = 84.92 \text{ kJ/kg}$$

$$\therefore \quad h_{m'} = h_l - 84.92 = 355.98 - 84.92 = 271.06 \text{ kJ/kg}$$

$$\therefore \quad m'\,(h_{m'} - h_{n'}) = (h_1 - h_{f_4})$$

or

$$m' = \frac{h_1 - h_{f_4}}{h_{m'} - h_{n'}} = \frac{2666.9}{271.06 - 29.98} = 11.06 \text{ kg}$$

$$(Q_1)_{total} = m'\,(h_l - h_{f_k}) + 1\,(h_1' - h_1)$$

[***At 15 bar, 300°C :*** h_g = 3037.6 kJ/kg, s_g = 6.918 kJ/kg K]

$$= 11.06 \times 326 + (3037.6 - 2789.9) = 3853.26 \text{ kJ/kg}$$

$$s_1' = 6.918 = s_2' = 0.423 + x_2' \times 8.052$$

$$\therefore \quad x_2' = \frac{6.918 - 0.423}{8.052} = 0.80.$$

$$h_2' = 121.5 + 0.807 \times 2432.9 = 2084.8 \text{ kJ/kg}$$

$$(W_T)_{steam} = h_1' - h_2' = 0.88\,(3037.6 - 2084.8) = 838.46 \text{ kJ/kg}$$

$$(W_T)_{total} = 11.06 \times 84.92 + 838.46 = 1777.67 \text{ kJ/kg}$$

Neglecting pump work,

$$\eta_{overall} = \frac{1777.67}{3853.26} = \mathbf{0.4613 \text{ or } 46.13\%}. \quad \textbf{(Ans.)}$$

GAS POWER CYCLES

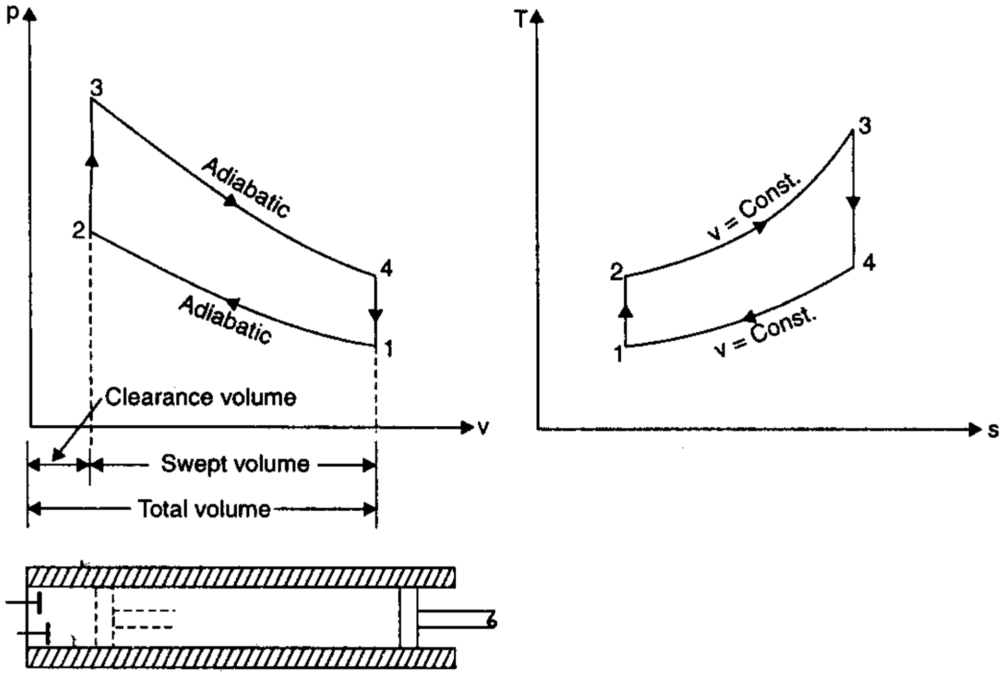
2.9. OTTO CYCLE

This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines.

Fig. 2.28 (*a*) and (*b*) shows the theoretical *p-V* diagram and *T-s* diagrams of this cycle respectively.

The point 1 represents that cylinder is full of air with volume V_1, pressure p_1 and absolute temperature T_1.

- Line 1-2 represents the *adiabatic compression* of air due to which p_1, V_1 and T_1 change to p_2, V_2 and T_2 respectively.
- Line 2-3 shows the *supply of heat to the air at constant volume* so that p_2 and T_2 change to p_3 and T_3 (V_3 being the same as V_2).
- Line 3-4 represents the *adiabatic expansion* of the air. During expansion p_3, V_3 and T_3 change to final values of p_4, V_4 or V_1 and T_4 respectively.
- Line 4-1 shows the *rejection of heat* by air at constant volume till original state (point 1) reaches.



(a) p-v diagram (b) T-s diagram

Fig. 2.28. Otto cycle.

Consider 1 kg of air (working substance) :

Heat supplied at constant volume $= c_v(T_3 - T_2)$.

Heat rejected at constant volume $= c_v\,(T_4 - T_1)$.

But, work done $= \text{Heat supplied} - \text{Heat rejected}$

$$= c_v (T_3 - T_2) - c_v (T_4 - T_1)$$

$$\therefore \quad \text{Efficiency} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v (T_3 - T_2) - c_v (T_4 - T_1)}{c_v (T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \qquad \text{...}(i)$$

Let compression ratio, $r_c\ (= r) = \frac{v_1}{v_2}$

and expansion ratio, $r_e\ (= r) = \frac{v_4}{v_3}$

(These two ratios are same in this cycle)

As $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$

Then, $T_2 = T_1 \cdot (r)^{\gamma - 1}$

Similarly, $\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1}$

or $T_3 = T_4 \cdot (r)^{\gamma - 1}$

Inserting the values of T_2 and T_3 in equation (*i*), we get

$$\eta_{otto} = 1 - \frac{T_4 - T_1}{T_4 \cdot (r)^{\gamma - 1} - T_1 \cdot (r)^{\gamma - 1}} = 1 - \frac{T_4 - T_1}{r^{\gamma - 1}(T_4 - T_1)}$$

$$= 1 - \frac{1}{(r)^{\gamma - 1}} \qquad \text{...(2.25)}$$

This expression is known as the ***air standard efficiency*** *of the Otto cycle.*

It is clear from the above expression that efficiency increases with the increase in the value of *r*, which means we can have maximum efficiency by increasing *r* to a considerable extent, but *due to practical difficulties its value is limited to about 8.*

The net **work done per kg** in the Otto cycle can also be expressed in terms of *pV*. If *p* is expressed in bar *i.e.*, 10^5 N/m^2, then work done

$$W = \left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}\right) \times 10^2 \text{ kJ} \qquad \text{...(2.26)}$$

Also $\frac{p_3}{p_4} = r^{\gamma} = \frac{p_2}{p_1}$

$$\therefore \quad \frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p$$

where r_p stands for *pressure ratio.*

and $v_1 = r v_2 = v_4 = r v_3 \qquad \left[\because \frac{v_1}{v_2} = \frac{v_4}{v_3} = r\right]$

$$\therefore \quad W = \frac{1}{\gamma - 1}\left[p_4 r\left(\frac{p_3 v_3}{p_4 r} - v_3\right) - p_1 r\left(\frac{p_2 v_2}{p_1 r} - v_2\right)\right]$$

$$= \frac{rv_2}{\gamma - 1}[p_4(r^{\gamma-1} - 1) - p_1(r^{\gamma-1} - 1)]$$

$$= \frac{v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(p_4 - p_1)]$$

$$= \frac{p_1v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(r_p - 1)] \qquad \text{...[2.26 (a)]}$$

Mean effective pressure is given by :

$$p_m = \left[\left(\frac{p_3v_3 - p_4v_4}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1}\right) \div (v_1 - v_2)\right] \text{bar}$$

Also

$$p_m = \frac{\left[\frac{p_1v_1}{\gamma - 1}(r^{\gamma-1} - 1)(r_p - 1)\right]}{(v_1 - v_2)}$$

$$= \frac{\frac{p_1v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(r_p - 1)]}{(v_1 - v_2)} = \frac{\frac{p_1v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(r_p - 1)]}{v_1 - \frac{v_1}{r}}$$

$$= \frac{\frac{p_1v_1}{\gamma - 1}[(r^{\gamma-1} - 1)(r_p - 1)]}{v_1\left(\frac{r - 1}{1}\right)}$$

i.e.

$$= \frac{p_1r[(r^{\gamma-1} - 1)(r_p - 1)]}{(\gamma - 1)(r - 1)}. \qquad \text{...(2.27)}$$

Example 2.15. *The efficiency of an Otto cycle is 60% and $\gamma = 1.5$. What is the compression ratio ?*

Solution. Efficiency of Otto cycle, $\eta = 60\%$

Ratio of specific heats, $\gamma = 1.5$

Compression ratio, $\mathbf{r} = ?$

Efficiency of Otto cycle is given by

$$\eta_{\text{Otto}} = 1 - \frac{1}{(r)^{\gamma-1}}$$

$$0.6 = 1 - \frac{1}{(r)^{1.5-1}}$$

or $$\frac{1}{(r)^{0.5}} = 0.4 \qquad \text{or} \qquad (r)^{0.5} = \frac{1}{0.4} = 2.5$$

or $$r = 6.25$$

Hence, *compression ratio* = **6.25. (Ans.)**

Example 2.16. *In a constant volume 'Otto cycle', the pressure at the end of compression is 15 times that at the start, the temperature of air at the beginning of compression is 38°C and maximum temperature attained in the cycle is 1950°C. Determine :*

(*i*) *Compression ratio.*

(*ii*) *Thermal efficiency of the cycle.*

(*iii*) *Work done per kg of air.*

Take γ *for air = 1.4.*

Solution. Refer Fig. 2.29.

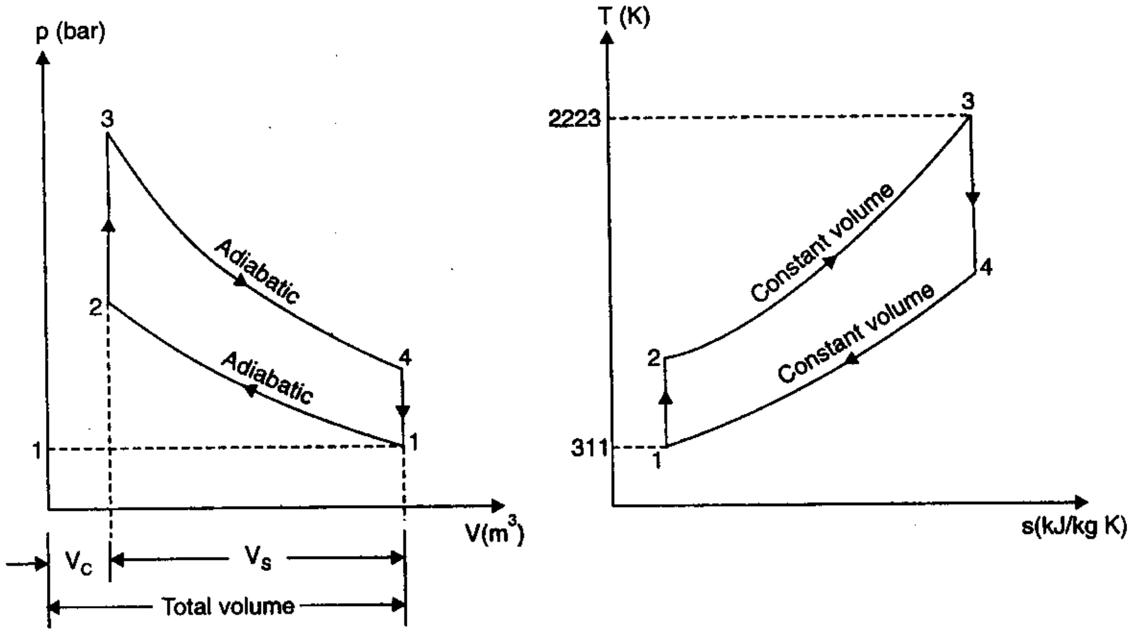


Fig. 2.29

Initial temperature, $T_1 = 38 + 273 = 311$ K

Maximum temperature, $T_3 = 1950 + 273 = 2223$ K.

(*i*) **Compression ratio, r :**

For adiabatic compression 1-2,

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \quad \text{or} \quad \left(\frac{v_1}{v_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

But $\frac{p_2}{p_1} = 15$ ...(given)

$\therefore$ $(r)^{\gamma} = 15$ $\left[\because \; r = \frac{v_1}{v_2}\right]$

or $(r)^{1.4} = 15$ or $r = (15)^{\frac{1}{1.4}} = (15)^{0.714} = 6.9$

Hence *compression ratio* = **6.9. (Ans.)**

(*ii*) **Thermal efficiency :**

Thermal efficiency, $\eta_{th} = 1 - \dfrac{1}{(r)^{\gamma-1}} = 1 - \dfrac{1}{(6.9)^{1.4-1}}$ = **0.538 or 53.8%. (Ans.)**

(*iii*) **Work done :**

Again, for adiabatic compression,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (6.9)^{1.4-1} = (6.9)^{0.4} = 2.16$$

or $$T_2 = T_1 \times 2.16 = 311 \times 2.16 = 671.7 \text{ K or } 398.7°\text{C}$$

For adiabatic expansion process 3-4

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = (r)^{\gamma-1} = (6.9)^{0.4} = 2.16$$

or $$T_4 = \frac{T_3}{2.16} = \frac{2223}{2.16} = 1029 \text{ K or } 756°\text{C}$$

Heat supplied per kg of air

$$= c_v(T_3 - T_2) = 0.717(2223 - 671.7) \qquad \left[c_v = \frac{R}{\gamma-1} = \frac{0.287}{1.4-1} = 0.717 \text{ kJ/ kg K}\right]$$

$= 1112.3$ kJ/kg or air

Heat rejected per kg of air

$= c_v(T_4 - T_1) = 0.717(1029 - 311)$

$= 514.8$ kJ/kg of air

∴ *Work done* = Heat supplied – heat rejected

= 1112.3 – 514.8

= **597.5 kJ or 597500 N-m. (Ans.)**

☞**Example 2.17.** *An engine working on Otto cycle has a volume of 0.45 m^3, pressure 1 bar and temperature 30°C at the beginning of compression stroke. At the end of compression stroke, the pressure is 11 bar. 210 kJ of heat is added during constant volume heating process. Determine :*

(*i*) *Pressures, temperatures and volumes at salient points in the cycle.*

(*ii*) *Percentage clearance.*

(*iii*) *Efficiency.*

(*iv*) *Net work per cycle.*

(*v*) *Mean effective pressure.*

(*vi*) *Ideal power developed by the engine if the number of working cycles per minute is 210.*

Assume the cycle is reversible.

Solution. Refer Fig. 2.30

Volume, $V_1 = 0.45\ m^3$

Initial pressure, $p_1 = 1$ bar

Initial temperature, $T_1 = 30 + 273 = 303$ K

Pressure at the end of compression stroke, $p_2 = 11$ bar

Heat added at constant volume = 210 kJ

Number of working cycles/min. = 210.

(*i*) **Pressures, temperatures and volumes at salient points :**

For adiabatic compression 1-2

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

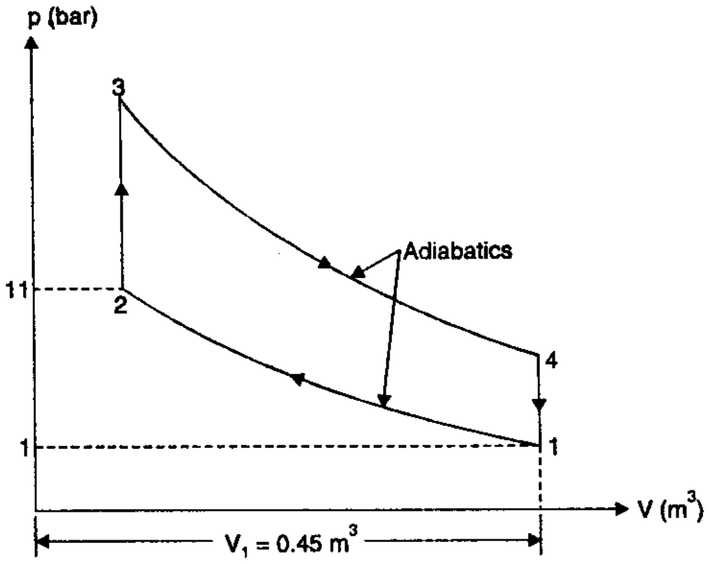


Fig. 2.30

or $$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma} \quad \text{or} \quad r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1}{1.4}} = (11)^{0.714} = 5.5$$

Also $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (5.5)^{1.4-1} = 1.977 \simeq 1.98$$

$\therefore$ $T_2 = T_1 \times 1.98 = 303 \times 1.98 =$ **600 K. (Ans.)**

Applying gas laws to points 1 and 2,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$\therefore$ $$V_2 = \frac{T_2}{T_1} \times \frac{p_1}{p_2} \times V_1 = \frac{600 \times 1 \times 0.45}{303 \times 11} = \mathbf{0.081\ m^3}. \quad \textbf{(Ans.)}$$

The heat supplied during the process 2-3 is given by :

$$Q_s = m\, c_v\, (T_3 - T_2)$$

where $$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.45}{287 \times 303} = 0.517 \text{ kg}$$

$\therefore$ $$210 = 0.517 \times 0.71\, (T_3 - 600)$$

or $$T_3 = \frac{210}{0.517 \times 0.71} + 600 = \mathbf{1172\ K.} \quad \textbf{(Ans.)}$$

For the constant volume process 2-3

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$\therefore \quad p_3 = \frac{T_3}{T_2} \times p_2 = \frac{1172}{600} \times 11 = \mathbf{21.48\ bar.} \quad \textbf{(Ans.)}$$

$$V_3 = V_2 = \mathbf{0.081\ m^3.} \quad \textbf{(Ans.)}$$

For the adiabatic (or isentropic) process 3-4

$$p_3V_3^{\gamma} = p_4V_4^{\gamma}$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma} = p_3 \times \left(\frac{1}{r}\right)^{\gamma}$$

$$= 21.48 \times \left(\frac{1}{5.5}\right)^{1.4} = \mathbf{1.97\ bar.} \quad \textbf{(Ans.)}$$

Also

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} = \left(\frac{1}{5.5}\right)^{1.4-1} = 0.505$$

$$\therefore \quad T_4 = 0.505\ T_3 = 0.505 \times 1172 = \mathbf{591.8\ K.} \quad \textbf{(Ans.)}$$

$$V_4 = V_1 = \mathbf{0.45\ m^3.} \quad \textbf{(Ans.)}$$

(*ii*) **Percentage clearance :**

Percentage clearance

$$= \frac{V_c}{V_s} = \frac{V_2}{V_1 - V_2} \times 100 = \frac{0.081}{0.45 - 0.081} \times 100$$

$$= \mathbf{21.95\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Efficiency :**

The heat rejected per cycle is given by

$$Q_r = mc_v(T_4 - T_1)$$

$$= 0.517 \times 0.71\,(591.8 - 303) = \mathbf{106\ kJ}$$

The air-standard efficiency of the cycle is given by

$$\eta_{otto} = \frac{Q_s - Q_r}{Q_s} = \frac{210 - 106}{210} = \mathbf{0.495\ or\ 49.5\%.} \quad \textbf{(Ans.)}$$

[It can also be calculated as follows :

$$\eta_{otto} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(5.5)^{1.4-1}} = \mathbf{0.495\ or\ 49.5\%.} \quad \textbf{(Ans.)}$$]

(*iv*) **Mean effective pressure, $\mathbf{p_m}$:**

The mean effective pressure is given by

$$p_m = \frac{W\ \text{(work done)}}{V_s\text{(swept volume)}} = \frac{Q_s - Q_r}{(V_1 - V_2)}$$

$$= \frac{(210 - 106) \times 10^3}{(0.45 - 0.081) \times 10^5} = \mathbf{2.818\ bar.} \quad \textbf{(Ans.)}$$

(*v*) **Power developed, P :**

Power developed, P = work done per second

= work done per cycle × number of cycles per second

= (210 – 106) × (210/60) = **364 kW. (Ans.)**

2.10. DIESEL CYCLE

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle, in, that heat is supplied at *constant pressure* instead of at constant volume. Fig. 2.31 (*a*) and (*b*) shows the *p-V* and *T-s* diagrams of this cycle respectively.

This cycle comprises of the following operations :

(*i*) 1-2......Adiabatic compression.
(*ii*) 2-3......Addition of heat at constant pressure.
(*iii*) 3-4......Adiabatic expansion.
(*iv*) 4-1......Rejection of heat at constant volume.

Point 1 represents that the cylinder is full of air. Let p_1, V_1 and T_1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (*i.e.* pV^{γ} = constant) till the values become p_2, V_2 and T_2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let volume increases from V_2 to V_3 and temperature T_2 to T_3, corresponding to point 3. This point (3) is called the *point of cut off*. The air then expands adiabatically to the conditions p_4, V_4 and T_4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.

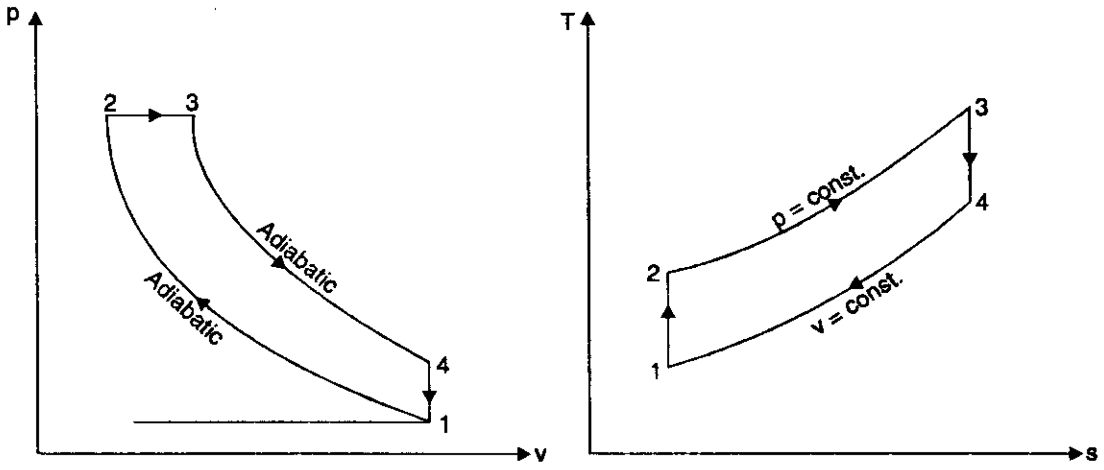


Fig. 2.31

Consider 1 kg of air.

Heat supplied at constant pressure $= c_p(T_3 - T_2)$

Heat rejected at constant volume $= c_v(T_4 - T_1)$

Work done = Heat supplied – Heat rejected

$= c_v(T_3 - T_2) - c_v(T_4 - T_1)$

$$\text{Air standar } \eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \qquad ...(i) \left[\because \frac{c_p}{c_v} = \gamma\right]$$

Let compression ratio, $r = \frac{v_1}{v_2}$ and cut off ratio, $\rho = \frac{v_3}{v_2}$ *i.e.* $\frac{\text{Volume at cut off}}{\text{Clearance volume}}$

Now, during adiabatic compression 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad \text{or} \quad T_2 = T_1 \,.\, (r)^{\gamma - 1}$$

During constant pressure process 2-3,

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho \quad \text{or} \quad T_3 = \rho \,.\, T_2 = \rho \,.\, T_1 \,.\, (r)^{\gamma - 1}$$

During adiabatic expansion 3-4

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma - 1} \qquad \left(\because \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = \frac{r}{\rho}\right)$$

$$\therefore \quad T_4 = \frac{T_3}{\left(\frac{r}{\rho}\right)^{\gamma - 1}} = \frac{\rho \,.\, T_1 (r)^{\gamma - 1}}{\left(\frac{r}{\rho}\right)^{\gamma - 1}} = T_1 \,.\, \rho^{\gamma}$$

By inserting values of T_2, T_3 and T_4 in equation (*i*), we get

$$\eta = 1 \; \frac{(T_1 \,.\, \rho^{\gamma} - T_1)}{(\rho \,.\, T_1 \,.\, (r)^{\gamma - 1} - T_1 \,.\, (r)^{\gamma - 1})} = 1 - \frac{(\rho^{\gamma} - 1)}{\gamma (r)^{\gamma - 1} (\rho - 1)}$$

$$= 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right] \qquad ...(2.28)$$

It may be observed that equation (2.28) for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $\rho > 1$. Hence *for a given compression ratio, the Otto cycle is more efficient.*

The *net work* for diesel cycle can be expressed in terms of *pv* as follows :

$$W = p_2(v_3 - v_2) + \frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_2 v_2 (\rho - 1) + \frac{p_3 \rho v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1}$$

$$= \frac{v_2 [p_2 (\rho - 1)(\gamma - 1) + p_2 \rho - p_4 r - (p_2 - p_1 r)]}{\gamma - 1} \qquad (\because \; p_2 = p_3)$$

$$= \frac{v_2\left[p_2(\rho - 1)(\gamma - 1) + p_2\left(\rho - \frac{p_4}{p_2} r\right) - p_2\left(1 - \frac{p_1 r}{p_2}\right)\right]}{\gamma - 1}$$

$$= \frac{p_1 v_1 r^{\gamma - 1}\left[(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} r^{1-\gamma} - (1 - r^{1-\gamma})\right]}{\gamma - 1}$$

$$= \frac{p_1 v_1 r^{\gamma - 1}\left[\gamma(\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)\right]}{(\gamma - 1)} \qquad \text{...(2.29)}$$

Mean effective pressure is given by

$$p_m = \frac{p_1 v_1 r^{\gamma - 1}\left[\gamma(\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)\right]}{(\gamma - 1) v_1 \left(\frac{r - 1}{r}\right)}$$

$$= \frac{p_1 r^{\gamma}\left[\gamma(\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)\right]}{(\gamma - 1)(r - 1)} \qquad \text{...(2.30)}$$

2.11. DUAL COMBUSTION CYCLE

This cycle (also called the *limited pressure cycle or mixed cycle*) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; *the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of laging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.*

The dual combustion cycle (Fig. 2.32) consists of the following operations :

(*i*) 1-2—Adiabatic compression

(*ii*) 2-3—Addition of heat at constant volume

(*iii*) 3-4—Addition of heat at constant pressure

(*iv*) 4-5—Adiabatic expansion

(*v*) 5-1—Rejection of heat at constant volume.

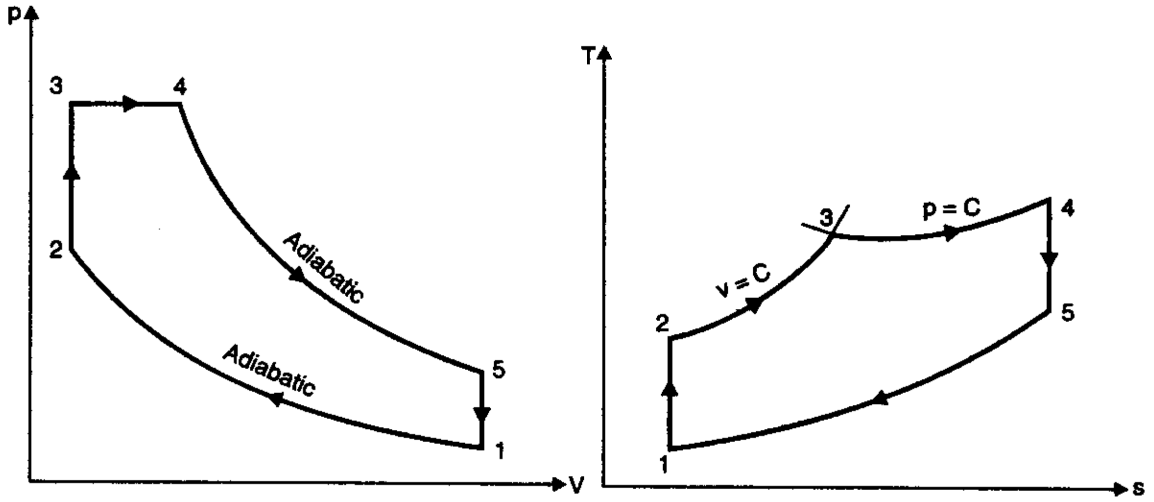


Fig. 2.32. Diesel cycle.

Consider 1 kg of air.

Total heat supplied = Heat supplied during the operation 2-3 + Heat supplied during the operation 3-4

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3)$$

Heat rejected during operation 5-1 $= c_v(T_5 - T_1)$

Work done = Heat supplied – Heat rejected

$$= c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)$$

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{c_v\,(T_3 - T_2) + c_p\,(T_4 - T_3) - c_v\,(T_5 - T_1)}{c_v\,(T_3 - T_2) + c_p\,(T_4 - T_3)}$$

$$= 1 - \frac{c_v\,(T_5 - T_1)}{c_v\,(T_3 - T_2) + c_p\,(T_4 - T_3)}$$

$$= 1 - \frac{c_v\,(T_5 - T_1)}{(T_3 - T_2) + \gamma\,(T_4 - T_3)} \qquad \ldots(i) \qquad \left(\because\ \gamma = \frac{c_p}{c_v}\right)$$

Compression ratio, $r = \dfrac{v_1}{v_2}$

During adiabatic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \qquad \ldots(ii)$$

During constant volume heating process

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

or $\dfrac{T_3}{T_2} = \dfrac{p_3}{v_2} = \beta$, where β is known as ***pressure or explosion ratio.***

or

$$T_2 = \frac{T_3}{\beta} \qquad \ldots(iii)$$

During adiabatic expansion process,

$$\frac{T_4}{T_5} = \left(\frac{v_5}{v_4}\right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma - 1} \qquad \ldots(iv)$$

$$\left(\because\ \frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{v_1}{v_2} \times \frac{v_2}{v_4} = \frac{v_1}{v_2} \times \frac{v_3}{v_4} = \frac{r}{\rho}, \rho \text{ being the cut off ratio}\right)$$

During constant pressure heating process,

$$\frac{v_3}{T_3} = \frac{v_4}{T_4}$$

$$T_4 = T_3\,\frac{v_4}{v_3} = \rho\,T_3 \qquad \ldots(v)$$

Putting the value of T_4 in the equation (*iv*), we get

$$\frac{\rho T_3}{T_5} = \left(\frac{r}{\rho}\right)^{\gamma - 1} \quad \text{or} \quad T_5 = \rho\,.\,T_3\left(\frac{\rho}{r}\right)^{\gamma - 1}$$

Putting the value of T_2 in equation (*ii*), we get

$$\frac{\dfrac{T_3}{\beta}}{T_1} = (r)^{\gamma - 1}$$

$$T_1 = \frac{T_3}{\beta} \cdot \frac{1}{(r)^{\gamma - 1}}$$

Now inserting the values of T_1, T_2, T_4 and T_5 in equation (*i*), we get

$$\eta = 1 - \frac{\left[\rho \, . \, T_3\left(\dfrac{\rho}{r}\right)^{\gamma - 1} - \dfrac{T_3}{\beta} \cdot \dfrac{1}{(r)^{\gamma - 1}}\right]}{\left[\left(T_3 - \dfrac{T_3}{\beta}\right) + \eta\,(\rho T_3 - T_3)\right]} = 1 - \frac{\dfrac{1}{(r)^{\gamma - 1}}\left(\rho^{\gamma} - \dfrac{1}{\beta}\right)}{\left(1 - \dfrac{1}{\beta}\right) + \gamma\,(\rho - 1)}$$

i.e.

$$\eta = 1 - \frac{1}{(r)^{\gamma - 1}} \cdot \frac{(\beta \, . \, \rho^{\gamma} - 1)}{[(\beta - 1) + \beta\gamma\,(\rho - 1)]} \qquad \text{...(2.31)}$$

Work done is given by,

$$W = p_3(v_4 - v_3) + \frac{p_4v_4 - p_5v_5}{\gamma - 1} - \frac{p_2v_2 - p_1v_1}{\gamma - 1}$$

$$= p_3v_3(\rho - 1) + \frac{(p_3\rho v_3 - p_5rv_3) - (p_2v_3 - p_1rv_3)}{\gamma - 1}$$

$$= \frac{p_3v_3\,(\rho - 1)(\gamma - 1) + p_4v_3\left(\rho - \dfrac{p_5}{p_4}r\right) - p_2v_3\left(1 - \dfrac{p_1}{p_2}r\right)}{\gamma - 1}$$

Also

$$\frac{p_3}{p_4} = \left(\frac{v_4}{v_3}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} \quad \text{and} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = r^{\gamma}$$

also,

$$p_3 = p_4,\ v_2 = v_3,\ v_5 = v_1$$

$\therefore$

$$W = \frac{v_3[p_3\,(\rho - 1)\,(\gamma - 1) + p_3\,(\rho - \rho^{\gamma}r^{1-\gamma}) - p_2\,(1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_2v_2[\beta\,(\rho - 1)\,(\gamma - 1) + \beta\,(\rho - \rho^{1-\gamma}) - (1 - r^{1-\gamma})]}{(\gamma - 1)}$$

$$= \frac{p_1(r)^{\gamma}v_1/\gamma[\beta\gamma\,(\rho - 1) + (\beta - 1) - r^{1-\gamma}\,(\beta\rho^{\gamma - 1})]}{\gamma - 1}$$

$$= \frac{p_1v_1r^{\gamma - 1}[\beta\gamma\,(\rho - 1) + (\beta - 1) - r^{\gamma - 1}(\beta\rho^{\gamma - 1})]}{\gamma - 1} \qquad \text{...(2.32)}$$

Mean effective pressure,

$$p_m = \frac{W}{v_1 - v_2} = \frac{W}{v_1\left(\dfrac{r - 1}{r}\right)} = \frac{p_1v_1[r^{1-\gamma}\,\beta\gamma\,(\rho - 1) + (\beta - 1) - r^{1-\gamma}\,(\beta\rho^{\gamma} - 1)]}{(\gamma - 1)\,v_1\left(\dfrac{r - 1}{r}\right)}$$

$$= \frac{p_1(r)^{\gamma}[\beta\,(\rho - 1) + (\beta - 1) - r^{1-\gamma}\,(\beta\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)} \qquad \text{...(2.33)}$$

DIESEL CYCLE

Example 2.18. *A diesel engine has a compression ratio of 15 and heat addition at constant pressure takes place at 6% of stroke. Find the air standard efficiency of the engine.*

Take γ for air as 1.4.

Solution. Refer Fig. 2.33.

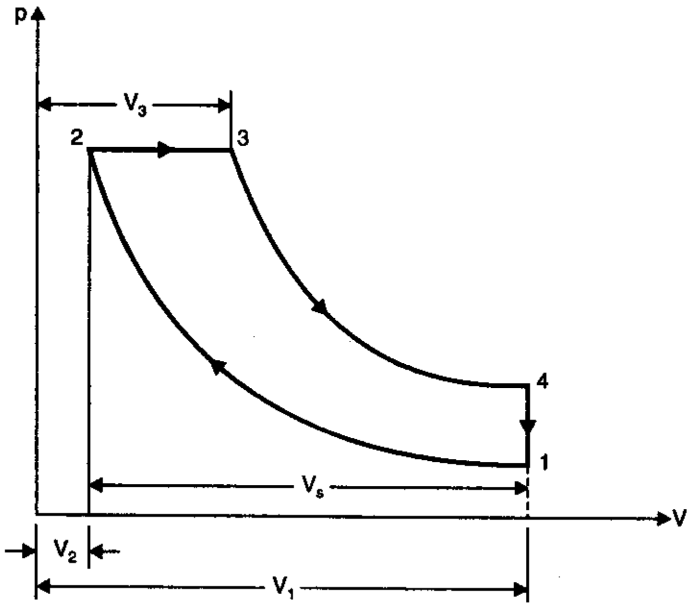


Fig. 2.33

Compression ratio, $r \left(= \frac{V_1}{V_2}\right) = 15$

γ for air $= 1.4$

Air standard efficiency of diesel cycle is given by

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right] \quad \text{...(i)}$$

where, ρ = Cut off ratio $= \frac{V_3}{V_2}$

But $$V_3 - V_2 = \frac{6}{100} V_s \quad (V_s = \text{stroke volume})$$

$$= 0.06\,(V_1 - V_2) = 0.06\,(15\,V_2 - V_2)$$

$$= 0.84\,V_2 \quad \text{or} \quad V_3 = 1.84\,V_2$$

$$\therefore \quad \rho = \frac{V_3}{V_2} = \frac{1.84\,V_2}{V_2} = 1.84$$

Putting the value in eqn. (*i*), we get

$$\eta_{\text{diesel}} = 1 - \frac{1}{1.4\,(15)^{1.4-1}}\left[\frac{(1.84)^{1.4}-1}{1.84-1}\right]$$

$$= 1 - 0.2417 \times 1.605 = \mathbf{0.612 \text{ or } 61.2\%}. \quad \textbf{(Ans.)}$$

Example 2.19. *The stroke and cylinder diameter of a compression ignition engine are 250 mm and 150 mm respectively. If the clearance volume is 0.0004 m^3 and fuel injection takes place at constant pressure for 5 percent of the stroke determine the efficiency of the engine. Assume the engine working on the diesel cycle.*

Solution. Refer Fig. 2.31.

Length of stroke, $L = 250 \text{ mm} = 0.25 \text{ m}$

Diameter of cylinder, $D = 150 \text{ mm} = 0.15 \text{ m}$

Clearance volume, $V_2 = 0.0004 \text{ m}^3$

Swept volume, $V_s = \pi/4\, D^2 L = \pi/4 \times 0.15^2 \times 0.25 = 0.004418 \text{ m}^3$

Total cylinder volume $= \text{Swept volume} + \text{Clearance volume}$

$= 0.004418 + 0.0004 = 0.004818 \text{ m}^3$

Volume at point of cut-off, $V_3 = V_2 + \frac{5}{100} V_s$

$= 0.0004 + \frac{5}{100} \times 0.004418 = 0.000621 \text{ m}^3$

$\therefore$ Cut-off ratio, $\rho = \frac{V_3}{V_2} = \frac{0.000621}{0.0004} = 1.55$

Compression ratio, $r = \frac{V_1}{V_2} = \frac{V_s + V_2}{V_2} = \frac{0.004418 + 0.0004}{0.0004} = 12.04$

Hence,
$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^\gamma - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4 \times (12.04)^{1.4 - 1}} \left[\frac{(1.55)^{1.4} - 1}{1.55 - 1} \right]$$

$= 1 - 0.264 \times 1.54 =$ **0.593 or 59.3%. (Ans.)**

Example 2.20. *Calculate the percentage loss in the ideal efficiency of a diesel engine with compression ratio 14 if the fuel cut-off is delayed from 5% to 8%.*

Solution. Let the clearance volume be unity.

Then, compression ratio, $r = 14$

Now, when the fuel is cut-off at 5%, we have

$$\frac{\rho - 1}{r - 1} = \frac{5}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = 0.05 \quad \text{or} \quad \rho - 1 = 13 \times 0.05 = 0.65$$

$\therefore$ $\rho = 1.65$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^\gamma - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4\,(14)^{1.4 - 1}} \left[\frac{(1.65)^{1.4} - 1}{1.65 - 1} \right]$$

$= 1 - 0.248 \times 1.563 = 0.612$ or 61.2%

When the fuel is cut-off at 8%, we have

$$\frac{\rho - 1}{r - 1} = \frac{8}{100} \quad \text{or} \quad \frac{\rho - 1}{14 - 1} = \frac{8}{100} = 0.08$$

$\therefore$ $\rho = 1 + 1.04 = 2.04$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^\gamma - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4\,(14)^{1.4 - 1}} \left[\frac{(2.04)^{1.4} - 1}{2.04 - 1} \right]$$

$= 1 - 0.248 \times 1.647 =$ **0.591 or 59.1%. (Ans.)**

Hence percentage loss in efficiency due to delay in fuel cut-off

$$= 61.2 - 59.1 = \mathbf{2.1\%}. \quad \textbf{(Ans.)}$$

Example 2.21. *The mean effective pressure of a Diesel cycle is 7.5 bar and compression ratio is 12.5. Find the percentage cut-off of the cycle if its initial pressure is 1 bar.*

Solution. Mean effective pressure, $p_m = 7.5$ bar

Compression ratio, $r = 12.5$

Initial pressure, $p_1 = 1$ bar

Refer Fig. 2.31.

The mean effective pressure is given by

$$p_m = \frac{p_1 r^\gamma [\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma - 1)(r - 1)}$$

$$7.5 = \frac{1 \times (12.5)^{1.4} [1.4(\rho - 1) - (12.5)^{1-1.4}(\rho^{1.4} - 1)]}{(1.4 - 1)(12.5 - 1)}$$

$$7.5 = \frac{34.33[1.4\rho - 1.4 - 0.364\rho^{1.4} + 0.364]}{4.6}$$

$$7.5 = 7.46\,(1.4\rho - 1.036 - 0.364\rho^{1.4})$$

$$1.005 = 1.4\rho - 1.036 - 0.364\rho^{1.4}$$

or $2.04 = 1.4\rho - 0.364\rho^{1.4}$ or $0.346\rho^{1.4} - 1.4\rho + 2.04 = 0$

Solving by trial and error method, we get

$$\rho = 2.24$$

$\therefore$ $\% \textit{ cut-off} = \dfrac{\rho - 1}{r - 1} \times 100 = \dfrac{2.24 - 1}{12.5 - 1} \times 100 = \mathbf{10.78\%}$. **(Ans.)**

☞**Example 2.22.** *An engine with 200 mm cylinder diameter and 300 mm stroke works on theoretical Diesel cycle. The initial pressure and temperature of air used are 1 bar and 27°C. The cut-off is 8% of the stroke. Determine :*

(*i*) *Pressures and temperatures at all salient points.*

(*ii*) *Theoretical air standard efficiency.*

(*iii*) *Mean effective pressure.*

(*iv*) *Power of the engine if the working cycles per minute are 380.*

Assume that compression ratio is 15 and working fluid is air.

Consider all conditions to be ideal.

Solution. Refer Fig. 2.34.

Cylinder diameter, $D = 200$ mm or 0.2 m

Stroke length, $L = 300$ mm or 0.3 m

Initial pressure, $p_1 = 1.0$ bar

Initial temperature, $T_1 = 27 + 273 = 300$ K

Cut-off $= \dfrac{8}{100} V_s = 0.08\, V_s$

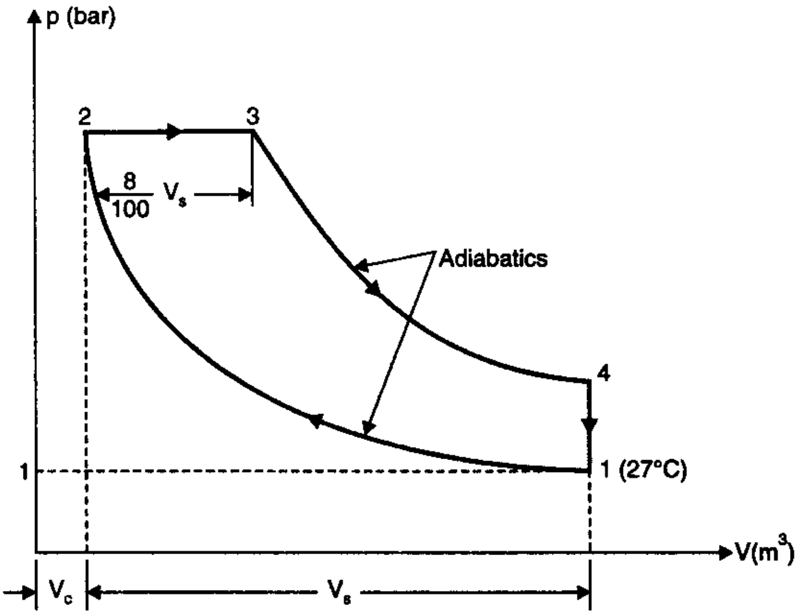


Fig. 2.34

(*i*) **Pressures and temperatures at salient points :**

Now, stroke volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.2^2 \times 0.3 = 0.00942\ \text{m}^3$

$$V_1 = V_s + V_c = V_s + \frac{V_s}{r-1} \qquad \left[\because\ V_c = \frac{V_s}{r-1}\right]$$

$$= V_s\left(1 + \frac{1}{r-1}\right) = \frac{r}{r-1} \times V_s$$

i.e.,

$$V_1 = \frac{15}{15-1} \times V_s = \frac{15}{14} \times 0.00942 = \mathbf{0.0101\ m^3}. \quad \textbf{(Ans.)}$$

Mass of the air in the cylinder can be calculated by using the gas equation

$$p_1V_1 = mRT_1$$

$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.0101}{287 \times 300} = 0.0117\ \text{kg/cycle}$$

For the adiabatic (or isentropic) process 1-2,

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (r)^{\gamma}$$

$\therefore$

$$p_2 = p_1 \cdot (r)^{\gamma} = 1 \times (15)^{1.4} = \mathbf{44.31\ bar.} \quad \textbf{(Ans.)}$$

Also,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1} = (15)^{1.4-1} = 2.954$$

$\therefore$

$$T_2 = T_1 \times 2.954 = 300 \times 2.954 = \mathbf{886.2\ K.} \quad \textbf{(Ans.)}$$

$$V_2 = V_c = \frac{V_s}{r-1} = \frac{0.00942}{15-1} = \mathbf{0.0006728\ m^3.}\quad \textbf{(Ans.)}$$

$$p_2 = p_3 = \mathbf{44.31\ bar.}\quad \textbf{(Ans.)}$$

% cut-off ratio $= \dfrac{\rho - 1}{r - 1}$

$$\frac{8}{100} = \frac{\rho - 1}{15 - 1}$$

i.e., $\rho = 0.08 \times 14 + 1 = 2.12$

$\therefore$ $V_3 = \rho\, V_2 = 2.12 \times 0.0006728 = \mathbf{0.001426\ m^3.}$ **(Ans.)**

$$\begin{bmatrix} V_3 \text{ can also be calculated as follows :} \\ V_3 = 0.08V_s + V_c = 0.08 \times 0.00942 + 0.0006728 = 0.001426\ \text{m}^3 \end{bmatrix}$$

For the constant pressure process 2-3

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

$$\therefore \quad T_3 = T_2 \times \frac{V_3}{V_2} = 886.2 \times \frac{0.001426}{0.0006728} = \mathbf{1878.3\ K.}\quad \textbf{(Ans.)}$$

For the isentropic process 3-4

$$p_3V_3^{\gamma} = p_4V_4^{\gamma}$$

$$p_4 = p_3 \times \left(\frac{V_3}{V_4}\right)^{\gamma} = p_3 \times \frac{1}{(7.07)^{1.4}}$$

$$= \frac{44.31}{(7.07)^{1.4}} = \mathbf{2.866\ bar.}\quad \textbf{(Ans.)}$$

$$\begin{bmatrix} \because\ \dfrac{V_4}{V_3} = \dfrac{V_4}{V_2} \times \dfrac{V_2}{V_3} = \dfrac{V_1}{V_2} \times \dfrac{V_2}{V_3} \\ = \dfrac{r}{\rho},\quad (\because\ V_4 = V_1) \\ = \dfrac{15}{2.12} = 7.07 \end{bmatrix}$$

Also, $\dfrac{T_4}{T_3} = \left(\dfrac{V_3}{V_4}\right)^{\gamma - 1} = \left(\dfrac{1}{7.07}\right)^{1.4 - 1} = 0.457$

$\therefore$ $T_4 = T_3 \times 0.457 = 1878.3 \times 0.457 = \mathbf{858.38\ K.}$ **(Ans.)**

$V_4 = V_1 = \mathbf{0.0101\ m^3.}$ **(Ans.)**

(*ii*) **Theoretical air standard efficiency :**

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma(r)^{\gamma - 1}}\left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right] = 1 - \frac{1}{1.4\,(15)^{1.4 - 1}}\left[\frac{(2.12)^{1.4} - 1}{2.12 - 1}\right]$$

$$= 1 - 0.2418 \times 1.663 = \mathbf{0.598\ or\ 59.8\%.}\quad \textbf{(Ans.)}$$

(*iii*) **Mean effective pressure, $\mathbf{p_m}$:**

Mean effective pressure of Diesel cycle is given by

$$p_m = \frac{p_1(r)^{\gamma}[\gamma(\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)}$$

$$= \frac{1 \times (15)^{1.4}[1.4(2.12 - 1) - (15)^{1-1.4}(2.12^{1.4} - 1)]}{(1.4 - 1)(15 - 1)}$$

$$= \frac{44.31[1.568 - 0.338 \times 1.863]}{0.4 \times 14} = \mathbf{7.424\ bar.}\quad \textbf{(Ans.)}$$

(*iv*) **Power of the engine, P :**

Work done per cycle $= p_m V_s = \dfrac{7.424 \times 10^5 \times 0.00942}{10^3} = 6.99$ kJ/cycle

Work done per second = Work done per cycle × No. of cycles per second

$= 6.99 \times 380/60 = 44.27$ kJ/s $= 44.27$ kW

Hence *power of the engine* = **44.27 kW. (Ans.)**

DUAL CYCLE

Example 2.23. *The swept volume of a diesel engine working on dual cycle is 0.0053 m*^{3} *and clearance volume is 0.00035 m*^{3}*. The maximum pressure is 65 bar. Fuel injection ends at 5 percent of the stroke. The temperature and pressure at the start of the compression are 80°C and 0.9 bar. Determine the air standard efficiency of the cycle. Take* γ *for air = 1.4.*

Solution. Refer Fig. 2.35.

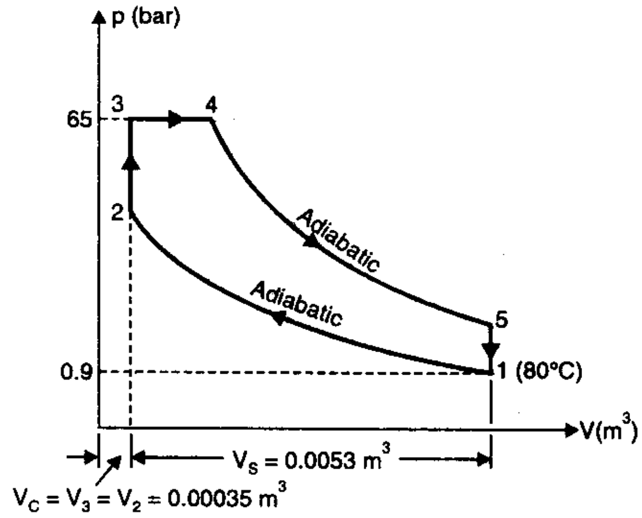


Fig. 2.35

Swept volume, $V_s = 0.0053 \text{ m}^3$

Clearance volume, $V_c = V_3 = V_2 = 0.00035 \text{ m}^3$

Maximum pressure, $p_3 = p_4 = 65$ bar

Initial temperature, $T_1 = 80 + 273 = 353$ K

Initial pressure, $p_1 = 0.9$ bar

η_{dual} **:**

The efficiency of a dual combustion cycle is given by

$$\eta_{\text{dual}} = 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{\beta \, . \, \rho^{\gamma} - 1}{(\beta - 1) + \beta\gamma(\rho - 1)} \right] \qquad ...(i)$$

Compression ratio, $r = \dfrac{V_1}{V_2} = \dfrac{V_s + V_c}{V_c} = \dfrac{0.0053 + 0.00035}{0.00035} = 16.14$

$[\because \; V_2 = V_3 = V_c = \text{clearance volume}]$

Cut-off-ratio,
$$\rho = \frac{V_4}{V_3} = \frac{\frac{5}{100}V_s + V_3}{V_3} = \frac{0.05\,V_3 + V_c}{V_c}$$
$$= \frac{0.05 \times 0.0053 + 0.00035}{0.00035} = 1.757 \text{ say } 1.76$$

Also during the compression operation 1-2,
$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

or
$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (16.15)^{1.4} = 49.14$$

or
$$p_2 = p_1 \times 49.14 = 0.9 \times 49.14 = 44.22 \text{ bar}$$

Pressure or explosion ratio, $\beta = \frac{p_3}{p_2} = \frac{62}{44.22} = 1.47$

Putting the value of r, ρ and β in equation (*i*), we get
$$\eta_{\text{dual}} = 1 - \frac{1}{(16.14)^{1.4-1}}\left[\frac{1.47 \times (1.76)^{1.4} - 1}{(1.47-1) + 1.47 \times 1.4\,(1.76-1)}\right]$$
$$= 7 - 0.328\left[\frac{3.243-1}{0.47+1.564}\right] = \mathbf{0.6383 \text{ or } 63.83\%.} \quad \textbf{(Ans.)}$$

2.12. GAS TURBINE CYCLES

Refer articles 5.11 and 5.12.

HIGHLIGHTS

1. Carnot cycle efficiency = $\frac{T_1 - T_2}{T_1}$.
2. Rankine cycle is the theoretical cycle on which steam prime movers work.

   Rankine efficiency = $\frac{h_1 - h_2}{h_1 - h_{f_4}}$.
3. Expression for efficiency of Rankine cycle in terms of temperatures,
$$\eta_{\text{Rankine}} = \frac{c_{pw}(T_1 - T_2)\left(1 + \frac{x_1 h_{fg_1}}{T_1}\right) - T_2 \cdot c_{pw} \log_e \frac{T_1}{T_2}}{c_{pw}\,(T_1 - T_2) + x_1 h_{fg_1}}.$$
4. Modified Rankine efficiency = $\frac{(h_1 - h_2) + (p_1 - p_2)\,v_2}{h_1 - h_{f_3}}$.
5. The theoretical thermal efficiency of reheat cycle is given by
$$\eta_{\text{thermal}} = \frac{[(h_1 - h_2) + (h_3 - h_4)] - W_p}{[(h_1 - h_{f_4}) + (h_3 - h_2)] - W_p}$$

   Pump work W_p is usually small and neglected.

6. $$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma - 1}}.$$

7. $$\eta_{Diesel} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right]$$

and $$p_m = \frac{p_1 r^{\gamma} [\gamma(\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)}.$$

8. $$\eta_{Dual} = 1 - \frac{1}{(r)^{\gamma - 1}} \frac{(\beta.\rho^{\gamma} - 1)}{[(\beta - 1) + \beta\gamma(\rho - 1)]}$$

$$p_m = \frac{p_1 (r)^{\gamma} [\beta(\rho - 1) + (\beta - 1) - r^{1-\gamma}(\beta\rho^{\gamma} - 1)]}{(\gamma - 1)(r - 1)}.$$

THEORETICAL QUESTIONS

1. Describe the different operations of Rankine-cycle. Derive also the expression for its efficiency.
2. How does a modified Rankine cycle differ from a Rankine cycle ? Write also the expression of modified Rankine efficiency.
3. State the methods of increasing the thermal efficiency of a Rankine cycle.
4. Explain with the help of neat diagrams a 'Regenerative Cycle'. Derive also an expression for its thermal efficiency.
5. State the advantages of regenerative cycle over simple Rankine cycle.
6. Explain with a neat diagram the working of a Binary vapour cycle.
7. Derive expressions for efficiencies of the following :
   (*i*) Diesel cycle (*ii*) Dual combustion cycle
   (*iii*) Open cycle (constant pressure) gas turbine.

UNSOLVED EXAMPLES

1. A simple Rankine cycle works between pressure of 30 bar and 0.04 bar, the initial condition of steam being dry saturated, calculate the cycle efficiency, work ratio and specific steam consumption.
   [Ans. 35%, 0.997, 3.84 kg/kWh]
2. A steam power plant works between 40 bar and 0.05 bar. If the steam supplied is dry saturated and the cycle of operation is Rankine, find :
   (*i*) Cycle efficiency (*ii*) Specific steam consumption.
   [Ans. (*i*) 35.5%, (*ii*) 3.8 kg/kWh]
3. Compare the Rankine efficiency of a high pressure plant operating from 80 bar and 400°C and a low pressure plant operating from 40 bar 400°C, if the condenser pressure in both cases is 0.07 bar.
   [Ans. 0.391 and 0.357]
4. A steam power plant working on Rankine cycle has the range of operation from 40 bar dry saturated to 0.05 bar. Determine :
   (*i*) The cycle efficiency (*ii*) Work ratio
   (*iii*) Specific fuel consumption. **[Ans. (*i*) 35.64%, (*ii*) 0.9957, (*iii*) 3.8 kg/kWh]**
5. In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 30 bar and the exhaust pressure is 0.25 bar. Determine :
   (*i*) The pump work (*ii*) Turbine work

(*iii*) Rankine efficiency (*iv*) Condenser heat flow

(*v*) Dryness at the end of expansion.

Assume flow rate of 10 kg/s. [**Ans.** (*i*) 30 kW, (*ii*) 7410 kW, (*iii*) 29.2%, (*iv*) 17900 kW, (*v*) 0.763]

6. In a regenerative cycle the inlet conditions are 40 bar and 400°C. Steam is bled at 10 bar in regenerative heating. The exit pressure is 0.8 bar. Neglecting pump work determine the efficiency of the cycle. [**Ans.** 0.296]

7. A turbine with one bleeding for regenerative heating of feed water is admitted with steam having enthalpy of 3200 kJ/kg and the exhausted steam has an enthalpy of 2200 kJ/kg. The ideal regenerative feed water heater is fed with 11350 kg/h of bled steam at 3.5 bar (whose enthalpy is 2600 kJ/h). The feed water (condensate from the condenser) with an enthalpy of 134 kJ/kg is pumped to the heater. It leaves the heater dry saturated at 3.5 bar. Determine the power developed by the turbine. [**Ans.** 16015 kW]

8. A binary-vapour cycle operates on mercury and steam. Saturated mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhausts at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.

(*i*) Find the overall efficiency of the cycle.

(*ii*) If 50000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?

(*iii*) Assuming that all processess are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?

(*iv*) If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler, and if the internal efficiencies of the mercury and steam turbines are 0.85 and 0.87 respectively, calculate the overall efficiency of the cycle. The properties of saturated mercury are as follows :

p (bar)	*t* (°C)	h_f	h_g	s_f	s_g	v_f	v_g
		(kJ/kg)		(kJ/kg K)		(m³/kg)	
4.5	450	62.93	355.98	0.1352	0.5397	79.9×10^{-6}	0.068
0.04	216.9	29.98	329.85	0.0808	0.6925	76.5×10^{-3}	5.178

[**Ans.** (*i*) 52.94%, (*ii*) 59.35 × 10^4 kg/h, (*iii*) 28.49 MW, (*iv*) 46.2%]

9. The efficiency of an Otto cycle is 50% and γ is 1.5. What is the compression ratio ? [**Ans.** 4]

10. An engine working on Otto cycle has a volume of 0.5 m³, pressure 1 bar and temperature 27°C at the commencement of compression stroke. At the end of compression stroke, the pressure is 10 bar. Heat added during the constant volume process is 200 kJ. Determine :

(*i*) Percentage clearance (*ii*) Air standard efficiency

(*iii*) Mean effective pressure

(*iv*) Ideal power developed by the engine if the engine runs at 400 r.p.m. so that there are 200 complete cycles per minutes. [**Ans.** (*i*) 23.76% ; (*ii*) 47.2% ; (*iii*) 2.37 bar (*iv*) 321 kW]

11. The compression ratio in an air-standard Otto cycle is 8. At the beginning of compression process, the pressure is 1 bar and the temperature is 300 K. The heat transfer to the air per cycle is 1900 kJ/kg of air. Calculate :

(*i*) Thermal efficiency (*ii*) The mean effective pressure.

[**Ans.** (*i*) 56.47% ; (*ii*) 14.24 bar]

12. An engine 200 mm bore and 300 mm stroke works on Otto cycle. The clearance volume is 0.0016 m³. The initial pressure and temperature are 1 bar and 60°C. If the maximum pressure is limited to 24 bar, find :

(*i*) The air-standard efficiency of the cycle (*ii*) The mean effective pressure for the cycle.

Assume ideal conditions. [**Ans.** (*i*) 54.08% ; (*ii*) 1.972 bar]

13. Calculate the air standard efficiency of a four stroke Otto cycle engine with the following data :

Piston diameter (bore) = 137 mm ; Length of stroke = 130 mm ;

Clearance volume 0.00028 m^3.

Express clearance as a percentage of swept volume. **[Ans.** 56.1% ; 14.6%]

14. In an ideal Diesel cycle, the temperatures at the beginning of compression, and the end of compression and at the end of the heat addition are 97°C, 789°C and 1839°C. Find the efficiency of the cycle. **[Ans.** 59.6%]

15. An air-standard Diesel cycle has a compression ratio of 18, and the heat transferred to the working fluid per cycle is 1800 kJ/kg. At the beginning of the compression stroke, the pressure is 1 bar and the temperature is 300 K. Calculate : (*i*) Thermal efficiency, (*ii*) The mean effective pressure. **[Ans.** (*i*) 61% ; (*ii*) 13.58 bar]

16. The following data belong to a Diesel cycle :
Compression ratio = 16 : 1 ; Heat added = 2500 kJ/kg ; Lowest pressure in the cycle = 1 bar ; Lowest temperature in the cycle = 27°C. Determine :
(*i*) Thermal efficiency of the cycle. (*i*) Mean effective pressure.
[Ans. (*i*) 45% ; (*ii*) 16.8 bar]

17. The compression ratio of an air-standard Dual cycle is 12 and the maximum pressure in the cycle is limited to 70 bar. The pressure and temperature of cycle at the beginning of compression process are 1 bar and 300 K. Calculate : (*i*) Thermal efficiency, (*ii*) Mean effective pressure.
Assume : cylinder bore = 250 mm, stroke length = 300 mm, c_p = 1.005, c_v = 0.718 and γ = 1.4.
[Ans. (*i*) 61.92% ; (*ii*) 9.847 bar]

18. The compression ratio of a Dual cycle is 10. The temperature and pressure at the beginning of the cycle are 1 bar and 27°C. The maximum pressure of the cycle is limited to 70 bar and heat supplied is limited to 675 kJ/kg of air. Find the thermal efficiency of the cycle. **[Ans.** 59.5%]

19. An air standard dual cycle has a compression ratio of 16, and compression begins at 1 bar, 50°C. The maximum pressure is 70 bar. The heat transferred to air at constant pressure is equal to that at constant volume. Determine :
(*i*) The cycle efficiency. (*ii*) The mean effective pressure of the cycle.
Take : c_p = 1.005 kJ/kg-K, c_v = 0.718 kJ/kg-K. **[Ans.** (*i*) 66.5% ; (*ii*) 4.76 bar]

20. In an air standard gas turbine engine, air at a temperature of 15°C and a pressure of 1.01 bar enters the compressor, where it is compressed through a pressure ratio of 5. Air enters the turbine at a temperature of 815°C and expands to original pressure of 1.01 bar. Determine the ratio of turbine work to compressor work and the thermal efficiency when the engine operates on ideal Brayton cycle.
Take : γ = 1.4, c_p = 1.005 kJ/kg K. **[Ans.** 2.393 ; 37.03%]

21. In an open cycle constant pressure gas turbine air enters the compressor at 1 bar and 300 K. The pressure of air after the compression is 4 bar. The isentropic efficiencies of compressor and turbine are 78% and 85% respectively. The air-fuel ratio is 80 : 1. Calculate the power developed and thermal efficiency of the cycle if the flow rate of air is 2.5 kg/s.

Take c_p = 1.005 kJ/kg K and γ = 1.4 for air and c_{pg} = 1.147 kJ/kg K and γ = 1.33 for gases. R = 0.287 kJ/kg K. Calorific value of fuel = 42000 kJ/kg. **[Ans.** 204.03 kW/kg of air ; 15.54%]

22. A gas turbine has a pressure ratio of 6/1 and a maximum cycle temperature of 600°C. The isentropic efficiencies of the compressor and turbine are 0.82 and 0.85 respectively. Calculate the power output in kilowatts of an electric generator geared to the turbine when the air enters the compressor at 15°C at the rate of 15 kg/s.

Take : c_p = 1.005 kJ/kg K and γ = 1.4 for the compression process, and take c_p = 1.11 kJ/kg K and η = 1.333 for the expansion process. **[Ans.** 920 kW]

23. The gas turbine has an overall pressure ratio of 5 : 1 and a maximum cycle temperature of 550°C. The turbine drives the compressor and an electric generator, the mechanical efficiency of the drive being 97%. The ambient temperature is 20°C and the isentropic efficiencies of the compressor and turbine are 0.8 and 0.83 respectively. Calculate the power output in kilowatts for an air flow of 15 kg/s. Calculate also the thermal efficiency and the work ratio.

Neglect changes is kinetic energy, and the loss of pressure in combustion chamber.

[Ans. 655 kW ; 12% ; 0.168]